ԵՐԵՎԱՆԻ ՊԵՏԱԿԱՆ ՀԱՄԱԼՍԱՐԱՆ

Տիգրան Հակոբյան

Դասական Նույնություններով Որոշվող Գերնույնությունները Կիսախմբերում

Ա.01.06 - "Հանրահաշիվ և թվերի տեսություն" մասնագիտությամբ

Ֆիզիկամաթեմատիկական գիտությունների թեկնածուի

Գիտական աստիճանի հայցման ատենախոսության

Սեղմագիր

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YEREVAN STATE UNIVERSITY

Tigran Hakobyan

Hyperidentities Defined By Classical Identities in Semigroups

Synopsis

Of dissertation for requesting the degree of candidate of

Physical and mathematical sciences specializing in

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Ատենախոսության թեման հաստատվել է ԵՊՀ մաթեմատիկայի և մեխանիկայի ֆակուլտետի խորհրդի կողմից:

Գիտական ղեկավար՝	ֆիզմաթ գիտությունների դոկտոր, պրոֆեսոր՝ Յու.Մ.Մովսիսյան
Պաշտոնական ընդդիմախոսներ՝	ֆիզմաթ գիտությունների դոկտոր, պրոֆեսոր՝ Հ.Բ.Մարանջյան
	ֆիզմաթ գիտությունների թեկնածու, դոցենտ՝ Ս.Ս.Դավիդով
Առաջատար կազմակերպություն՝	Մոսկվայի Պետական Մանկավարժական Համալսարան

Պաշտպանությունը կկայանա 2017թ. հունվարի 27-ին ժ. 15:00-ին Երևանի Պետական Համալսարանում գործող ԲՈՀ-ի 050 մասնագիտական խորհրդի նիստում (0025, Երևան, Ալեք Մանուկյան 1):

Ատենախոսությանը կարելի է ծանոթանալ Երևանի Պետական Համալսարանի գրադարանում:

Սեղմագիրն առաքված է 2016թ. դեկտեմբերի 26-ին:

Մասնագիտական խորհիրդի գիտական քարտուղար՝ Տ.Ն.Հարությունյան

The topic of dissertation approved at a meeting of academic council of the faculty of Mathematics and Mechanics of the Yerevan State University.

Scientific adviser:	Doctor of Phys. math sciences, professor: Yu.M.Movsisyan
Official opponents:	Doctor of Phys. math sciences, professor: H.B.Maranjian
	Candidate of Phys. math sciences, docent: S.S.Davidov
Leading institution:	Moscow State Pedagogical University

Dissertation defense will take place on January 27, 2017 at 15:00, during the specialized meeting of the Higher Attestation Commission council 050 at YSU (1 Alex Manoogian, Yeravan 0025, Armenia).

The dissertation is available in the library of Yerevan State University.

The Synopsis was sent on December 26, 2016.

Scientific secretary of specialized council T.N.Harutyunyan

General characterization of the work

Actuality of the Problem

About the second order formulas (and languages) see [1, 2]. Recall ([3, 4, 7, 20, 21, 5]) that a *hyperidentity* is a second order formula of the following form:

$$\forall X_1, \ldots, X_m \forall x_1, \ldots, x_n \quad (\omega_1 = \omega_2), \tag{$*$}$$

where ω_1, ω_2 are words(terms) in the alphabet of functional variables $X_1, \ldots, X_m$ and of object variables $x_1, \ldots, x_n$. However, hyperidentities are usually presented without universal quantifiers, i.e. in the form: $\omega_1 = \omega_2$. A hyperidentity $\omega_1 = \omega_2$ is said to be *satisfied* in an algebra $(Q; \Sigma)$ (or the algebra $(Q; \Sigma)$ *satisfies* the hyperidentity $\omega_1 = \omega_2$) if this equality is true when any functional variable X_i is replaced by any operation of the same arity from Σ (the possibility of such replacements is assumed) and any object variable x_j is replaced by any element of Q. This concept was first considered in [6] for algebras with binary quasigroup operations.

There are different groups of individuals considering different versions of hyperidentities depending on the domain of interpretation (see [18]). Namely,

- Students of W. Neumann and W. Taylor (as in [24]) will fix a similarity type τ , say $\tau =< 2, 2, 3, 1 >$ has symbols for two binary operations and a symbol for a unary and another ternary operation. Σ is then the collection of all terms formed from these fundamental operations, including the projection operations which appear as single variables.

- Students of K. Denecke, D. Schweigert, et.al. have a notion of hypersubstitution which is covered in [14]; an effect is that certain varieties of hyperidentities can be seen as having Σ range over certain subsets of the set of terms. In particular, prehyperidentities will exclude the projection terms, and M-hyperidentities are a subset which are derived from a monoid M of hypersubstitutions.

- Students of V. Belousov and Y. Movsisyan use the convention (e.g. in [6],

[23]) that Σ is a set of fundamental operations. This is in the context of studying structures with finitely many operations $\cdot, \odot, \ldots$ Some papers have these as binary quasigroup operations on the same set, and the hyperidentity is a relation involving only these members of Σ. We are going to concentrate on this type of interpretation for semigroups.

For example ([22, 23, 8]), in the term algebra of any Boolean algebra the following hyperidentity is satisfied:

$$X(x_1, \ldots, x_{n-1}, X(x_1, \ldots, x_{n-1}, X(x_1, \ldots, x_n))) = X(x_1, \ldots, x_n),$$

for any positive integer n. For the two-element Boolean algebra this hyperidentity means the equivalence of the corresponding two switching circuits. Varieties of varieties are characterized by hyperidentities ([24]). For applications of hyperidentities see [25, 26, 27, 28, 29, 31, 32, 33].

The variety V satisfies the given hyperidentity if every algebra of the variety V satisfies the same hyperidentity. This hyperidentity is called hyperidentity of the variety V.

The hyperidentity $(*)$ is said to be *non-trivial* if $m > 1$, and it is *trivial* if $m = 1$. The number m is called the *functional rank* of the hyperidentity $(*)$.

Hyperidentities of *associativity* are built on the equation of associativity: $x(yz) = (xy)z$. For example:

$$X(x, Y(y, z)) = Y(Z(x, y), z)$$

is a non-trivial hyperidentity of associativity with functional rank 3.

A binary algebra $(Q; \Sigma)$ is said to be a q-algebra (e-algebra) if there is an operation $A \in \Sigma$ such that $Q(A)$ is a quasigroup (a groupoid with a unit). A binary algebra $(Q; \Sigma)$ is called *functionally non-trivial* if $|\Sigma| > 1$. It is known ([3, 4]) (see also [8, 7]) that if an associative non-trivial hyperidentity is satisfied in a functionally non-trivial q-algebra (e-algebra), then this hyperidentity can only be of functional rank 2 and of one in the following forms:

$$X(x, Y(y,z)) = Y(X(x,y),z), \tag{1}$$

$$X(x, Y(y,z)) = X(Y(x,y),z), \tag{2}$$

$$Y(x, Y(y,z)) = X(X(x,y),z). \tag{3}$$

Moreover, in the class of q-algebras (e-algebras) the hyperidentity (3) implies the hyperidentity (2), and the hyperidentity (2) implies the hyperidentity (1).

Let $Q(\cdot)$ be a semigroup. The following function is said to be its *binary polynomial* (term):

$$F(x,y) = z_1^{\varepsilon_1} z_2^{\varepsilon_2} \dots z_n^{\varepsilon_n}\,, \tag{4}$$

where $n \in \mathbb{N}, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n \in \mathbb{N}$, $z_1, z_2, \dots, z_n \in \{x,y\}$ and $z_i \neq z_{i+1}$. The number n is called the length of this representation of the polynomial $F(x,y)$. However, due to the identities in the semigroup $Q(\cdot)$, the same polynomial $F(x,y)$ can have different representations of the form (4).

By Q^2_{pol} we denote the collection of all binary polynomials of the semigroup $Q(\cdot)$.

We say that the hyperidentity $(*)$ is *polynomially satisfied* (valid) in the semigroup $Q(\cdot)$ if this hyperidentity is satisfied in the binary algebra $\left(Q; Q^2_{pol}\right)$. In [9], it is proved that the class of all semigroups polynomially satisfying the trivial associative hyperidentity:

$$X(x, X(y,z)) = X(X(x,y),z)\,, \tag{$*,*$}$$

forms a finitely based variety of semigroups, and the basis of this variety contains about 1000 identities. These semigroups are called *hyperassociative*. For example, any two-element semigroup is hyperassociative [9]. In [10] (see also [11]), a basis of the identities of the same variety is given, which contains the following four identities:

$$x^4 = x^2,$$

$$xyxzxyx = xyzyx,$$

$$xy^2z^2 = xyz^2yz^2,$$

$$x^2y^2z = x^2yx^2yz,$$

i.e. a semigroup is hyperassociative iff it satisfies these four identities (see also [12, 18, 13, 14, 15]).

We also consider other algebras. For this we need the following definitions:

Definition 0.1. *The polynomial $F(x, y)$ depends on the variable x essentially in the semigroup $Q(\cdot)$ if there are elements $x_1, x_2, y \in Q$ such that $F(x_1, y) \neq F(x_2, y)$. In the same way the essentially dependence of the polynomial $F(x, y)$ on the variable y is defined.*

Definition 0.2. *The polynomial $F(x, y)$ is called essential if it depends on both variables x and y essentially.*

We also let Q^2_{epol} be the collection of all binary essential polynomials of the semigroup $Q(\cdot)$.

We say that the hyperidentity $(*)$ is *essentially satisfied* (valid) or is *satisfied for essential polynomials* in the semigroup $Q(\cdot)$ if this hyperidentity is satisfied in the binary algebra $\left(Q; Q^2_{epol}\right)$.

Definition 0.3. *We say that two hyperidentities are* equivalent *(written as $\Leftrightarrow$), if they simultaneously are either polynomially satisfied or none of them is polynomially satisfied in any semigroup $Q(\cdot)$. It is said that the hyperidentity (h_1) implies the hyperidentity (h_2), written as $(h_1) \Rightarrow (h_2)$, if in all semigroups where the hyperidentity (h_1) is satisfied polynomially, the hyperidentity (h_2) is also satisfied polynomially.*

Likewise, we also define the following:

Definition 0.4. *We say that two hyperidentities are* essentially equivalent *(written as* $\Leftrightarrow_e$*), if they simultaneously are either essentially satisfied or none of them is essentially satisfied in any semigroup* $Q(\cdot)$*. It is said that the hyperidentity* (h_1) *essentially implies the hyperidentity* (h_2)*, written as* $(h_1) \Rightarrow_e (h_2)$*, if in all semigroups where the hyperidentity* (h_1) *is satisfied essentially, the hyperidentity* (h_2) *is also satisfied essentially.*

The Aim and Objectives of the Dissertation

The main aims of the present thesis are the followings:

1. Investigate the semigroups satisfying associative hyperidentities and find finite bases for variety of those semigroups.

2. Investigate the semigroups satisfying distributive hyperidentities and find finite bases for variety of those semigroups.

3. Investigate the semigroups satisfying alternative hyperidentities and find finite bases for variety of those semigroups.

4. Prove an Artin type theorem for hyperalternative semigroups.

5. Investigate the semigroups satisfying transitive hyperidentities and find finite bases for variety of those semigroups.

The Object of the Investigation

The objects of this investigation are the identities of semigroups, second order formulas, associative hyperidentities, distributive hyperidentities, alternative hyperidentities, transitive hyperidentities, polynomial and essential satisfiability.

The Methods of the Investigation

In this thesis we used the results of the word problem, semigroup theory and universal algebraic theory, as well as, the concepts of equivalence and essential equivalence of hyperidentities.

Scientific Innovation

Following types of results are obtained in the thesis:

1. Explicit description of the necessary and sufficient conditions of a semigroup to polynomially or essentially satisfy an associative hyperidentity of functional ranks 1, 2, 3 or 4.

2. Explicit description of the necessary and sufficient conditions of a semigroup to essentially satisfy a distributive hyperidentity of functional ranks 1, 2, 3, 4 or 5.

3. Explicit description of the necessary and sufficient conditions of a semigroup to essentially satisfy an alternative hyperidentity.

4. Prove an Artin type theorem about hyperalternative semigroups.

5. Explicit description of the necessary and sufficient conditions of a semigroup to polynomially or essentially satisfy a transitive hyperidentity of functional ranks 1, 2, 3 or 4.

6. Define equivalence and essential equivalence of hyperidentities and investigate equivalence and essential equivalence of hyperidentities.

All of the main results are new.

Theoretical and Practical Value

Associative, distributive, alternative and transitive hyperidentities of semigroups are some of the most natural second order properties of semigroups. These properties have many applications in various fields, such as: semigroup theory, word problem in semigroup, universal algebra, hypersubstitution theory, clones theory, graph theory and circuit optimization.

The Approbation of the Results

The main results of the thesis have been presented to the following international scientific conferences and scientific seminars:

- "Hyperidentities in semigroups", Yerevan State University, Algebra Seminar, May, 2016, Yerevan, Armenia
- "Hyperidentities in semigroups", University of Illinois Urbana-Champaign, Logic Seminar, November, 2015, Urbana, USA
- "Hyperalternative semigroups," International Conference Maltsev Meetings, Collection of Abstracts, p. 95, October 11-14, 2011, Novosibirsk, Russia.
- "The semigroups with ternary alternative hyperidentities," Computer Science and Information Technologies, p. 55-57, September 26-30, 2011, Yerevan, Armenia.

Publications

The main results of the thesis were published in four scientific articles in the journals which we bring at the end of the Synopsis.

The Structure and Volume of the Thesis

The thesis is consisted of five Chapters, Conclusion, Index and a list of References. The publications of the author are four articles. The number of references is 36. The volume of the thesis is 101 pages.

The Main Content of the Thesis

This thesis consists of five chapters and in each chapter we consider hyperidentities of some type. More specifically here are the descriptions of the chapters:

Chapter 1: Introduction

This chapter introduces some basic concepts which we need in the thesis. In the introduction, we introduce some concepts from universal algebra, semigroup theory which we use throughout the thesis. Furthermore, we describe the problems we aim to solve and give some examples to further ease the understanding of the concepts.

Chapter 2: Associative Hyperidentities

In the second chapter, we consider the hyperidentities which are built on the equation of associativity: $x(yz) = (xy)z$ in semigroups.

We first consider the case of satisfiability for the essential polynomials. We present necessary and sufficient conditions for each of the associative hyperidentities of rank 2, 3 and 4 to be essentially satisfied in a semigroup $Q(\cdot)$. As a consequence, we prove that, in difference to the case of associative trivial hyperidentity, the class of all semigroups, in which the nontrivial associative hyperidentity is essentially satisfied, is a finite union of the finitely based varieties of semigroups. As a result, the nontrivial associative hyperidentities are classified up to essential satisfiability. This section is also covered in [13].

Next, we consider the case of satisfiability for all polynomials. We prove that if a semigroup polynomially satisfies to an associative hyperidentity of rank 2, 3 or 4, then it has to be trivial, that is, the one element semigroup.

This chapter together with [9], [10] and [12] fully cover all the cases of the associative hyperidentities in semigroups.

Here is the main result of this chapter:

Theorem 2.3.9. *Any non-trivial associative hyperidentity is essentially equivalent to one of following hyperidentities:*

$$X\left(X\left(x,y\right),z\right) = X\left(x,Y\left(y,z\right)\right) \tag{5}$$

$$X\left(X\left(x,y\right),z\right)=Y\left(x,X\left(y,z\right)\right) \tag{6}$$

$$X\left(X\left(x,y\right),z\right)=Y\left(x,Y\left(y,z\right)\right) \tag{7}$$

$$X\left(Y\left(x,y\right),z\right)=X\left(x,Y\left(y,z\right)\right) \tag{8}$$

$$X\left(Y\left(x,y\right),z\right)=Y\left(x,X\left(y,z\right)\right) \tag{9}$$

Moreover, we have the following implications: (5) $\Rightarrow_e$ (7), (5) $\Rightarrow_e$ (6) $\Rightarrow_e$ (9) $\Rightarrow_e$ (8).

Chapter 3: Distributive Hyperidentities

In the third chapter, we consider the hyperidentities which are built on the equation of distributivity: $x(yz) = (xy)(xz)$ in semigroups.

Again, we consider the case of satisfiability for the essential polynomials. We present necessary and sufficient conditions for each of the distributive hyperidentities of rank 1, 2, 3, 4 or 5 to be essentially satisfied in a semigroup $Q(\cdot)$. As a consequence, we prove that, the class of all semigroups in which a distributive hyperidentity is essentially satisfied is a finite union of the finitely based varieties of semigroups. This section is also covered in [19].

Note that, the case of satisfiability of the distributive hyperidentities for all polynomials has already been considered in [36].

Thus, this chapter with [36] also fully covers all the cases of the distributive hyperidentities in semigroups.

Here is the main result of this chapter:

Theorem 3.2.18. *Any (left) distributive hyperidentity is essentially equivalent either to the hyperidentity*

$$X(x,X(y,z))=X(X(x,y),X(x,z)) \tag{10}$$

whose rank is one or to the hyperidentity

$$X(x,X(y,z))=X(X(x,y),Y(x,z)) \tag{11}$$

whose rank is two. Moreover, we have (11) $\Rightarrow_e$ (10)*.*

Chapter 4: Alternative Hyperidentities

In the fourth chapter we consider the alternative hyperidentities in semigroups, which are described below.

The following hyperidentities are consequences of the trivial associative hyperidentity $(*,*)$:

$$X(x, X(x, z)) = X(X(x, x), z)\,,$$

$$X(x, X(y, y)) = X(X(x, y), y)\,,$$

which are called *left* and *right alternative* hyperidentities, respectively.

The equivalence of the left and right alternative hyperidentities is evident. Indeed, the binary polynomial F of the semigroup $Q(\cdot)$ satisfies the identity:

$$F(F(x, x), y) = F(x, F(x, y))$$

iff the binary polynomial $F^*(x, y) = F(y, x)$ satisfies the identity:

$$F^*(F^*(y, x), x) = F^*(y, F^*(x, x)).$$

Moreover, $F = (F^*)^*$.

Thus, we call a semigroup *hyperalternative* if the left (or right) alternative hyperidentity is polynomially satisfied in this semigroup.

In non-associative ring theory, the Artin theorem states that in an alternative algebra the subalgebra generated by any two elements is associative (see [16]). Here, we consider *hyperalternative* (cf. [17]) and *hyperassociative* (see [9, 10, 11, 12, 18, 13, 19]) semigroups and prove that any two elements in a hyperalternative semigroup generate a hyperassociative subsemigroup.

Here is the main result of this chapter:

Theorem 4.2.1. *If the semigroup $Q(\cdot)$ is hyperalternative, then any two elements in $Q(\cdot)$ generate a hyperassociative subsemigroup, i.e. the following identity*

$$X(X(A(x,y),B(x,y)),C(x,y)) = X(A(x,y),X(B(x,y),C(x,y))),$$

holds for any binary polynomials X, A, B, C of $Q(\cdot)$, that is the semigroup $Q(\cdot)$ polynomially satisfies the following hyperidentity of the functional rank 4:

$$X(X(Y(x,y),Z(x,y)),U(x,y)) = X(Y(x,y),X(Z(x,y),U(x,y))).$$

Chapter 5: Transitive Hyperidentities

Finally, in chapter five, we consider the transitive hyperidentities which are built on the equation of transitivity: $(xy)(yz) = xz$ in semigroups.

We consider the case of satisfiability for all polynomials. We explicitly present necessary and sufficient conditions for the transitive hyperidentities to be polynomially satisfied in a semigroup $Q(\cdot)$. As a consequence, we get that if a non-trivial semigroup satisfies a transitive hyperidentity, then it has to be one of functional rank 1. This work is partially covered in [35].

Thus, this work also fully covers all the cases of the transitive hyperidentities in semigroups that are polynomially satisfied. Here is the main result of this chapter:

Theorem 5.2.1. *Any transitive hyperidentity is polynomially equivalent either to the hyperidentity*

$$X(X(x,y),X(y,z)) = X(x,z), \tag{12}$$

or to the hyperidentity

$$X(X(x,y),X(y,z)) = Y(x,z). \tag{13}$$

Moreover, we have (13) $\Rightarrow$ (12).

One can easily adapt the techniques in this thesis to characterize all the semigroups where the transitive hyperidentities are essentially satisfied.

List of Publications of the Author in the Journals

1. Yu.M.Movsisyan and T.A.Hakobyan, *Associative nontrivial hyperidentities in semigroups*, Journal of Contemporary Mathematical Analysis, vol. 46, 3(2011), 121-130, link.springer.com/article/10.3103/S1068362311030010.

2. Yu.M.Movsisyan and T.A.Hakobyan, *Distributive hyperidentities in semigroups*, J. Contemp. Math. Anal. 46(6) (2011) 293-298, http://link.springer.com/article/10.3103/S1068362311060021.

3. T.A.Hakobyan and Yu.M.Movsisyan, *Artin theorem for semigroups* Journal of Algebra and its Applications, 2016 (Online Ready), http://dx.doi.org/10.1142/S0219498817500347.

4. T.A.Hakobyan, *Transitive hyperidentity in semigroups*, Proceedings of Yerevan State University, 3 (2016), 52-55.

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Ամփոփում

Այս թեզում մենք հետազոտել ենք պարբեր գերնույնություններ կիսախմբերում: Յուրաքանչյուր քննարկված գերնույնության համար բացահայտ նկարագրվել են կիսախմբի անհրաժեշտ և բավարար պայմանները՝ տալով վերջավոր թվով նույնություններ, որտեղ տվյալ գերնույնությունը բազմանդամորեն կամ էապես բավարարվում է: Հետևաբար, մենք ապացուցում ենք, որ բոլոր քննարկված գերնույնությունների դեպքում հանգում ենք, կա՛մ վերջավոր բազիսով կիսախմբերի բազմաձևությունների, կա՛մ էլ՝ վերջավոր բազիսով կիսախմբերի բազմաձևությունների վերջավոր միավորման:

Ավելի ճշգրիտ՝ հետազոտել ենք կիսախմբի անհրաժեշտ և բավարար պայմանները, որպեսզի այնտեղ մեկից մեծ ֆունկցիոնալ ռանգով զուգորդական գերնույնությունները բազմանդամորեն կամ էապես բավարարվեն: Այս աշխատանքն ամբողջությամբ ապառում է զուգորդական ոչ–տրիվյալ գերնույնությունների բոլոր դեպքերը կիսախմբերում:

Մենք նաև հետազոտել ենք կիսախմբի անհրաժեշտ և բավարար պայմանները, որպեսզի այնտեղ բոլոր հնարավոր ֆունկցիոնալ ռանգով բաշխական գերնույնությունները էապես բավարարվեն: Այս աշխատանքը [36] աշխատանքի հետ մեկտեղ ամբողջությամբ ապառում է բաշխական գերնույնությունների բոլոր դեպքերը կիսախմբերում:

Այնուհետև, կենտրոնացել ենք գերզուգորդական կիսախմբերի դասի մի հետաքրքիր ընդլայնման՝ գերալտերնատիվ կիսախմբերի դասի վրա: Ապացուցվել է հետևյալ Արթինի տիպի թեորեմը. գերալտերնատիվ կիսախմբի ցանկացած 2 տարր ծնում են գերզուգորդական ենթակիսախումբ: Որպես հետևանք, նաև ստացել ենք կիսախմբի անհրաժեշտ և բավարար պայմաններ, որպեսզի այնտեղ բազմանդամորեն բավարարվի ալտերնատիվ գերնույնությունը:

Վերջում, մենք ստացել ենք կիսախմբի անհրաժեշտ և բավարար պայմանները, որպեսզի բոլոր հնարավոր ֆունկցիոնալ ռանգով փոխանցական գերնույնությունները բազմանդամորեն բավարարվեն կիսախմբում: Նմանատիպ արդյունքներ տեղի ունեն նաև այն դեպքում, երբ փոխանցական գերնույնությունները բավարարվում են էապես: Այս արդյունքները նույնպես ամբողջությամբ սպառում են փոխանցական գերնույնությունների բոլոր դեպքերը կիսախմբերում:

Заключение

В этой диссертации мы рассмотрели сверхтождества в полугруппах. Для каждого данного сверхтождества мы даем явное описание необходимых и достаточных условий для полугрупп, давая конечное число тождеств - для которых оно удовлетворяет либо полиномиально либо существенно. Таким образом мы доказываем, что класс всех полугрупп, в которых выполняется одно из расмотренных сверхтождеств полиномиально или существенно, есть либо конечно-базируемое многообразие полугрупп либо конечное объединение конечно-базируемых многообразий полугрупп.

Более точно, мы исследовали необходимые и достаточные условия для полугрупп, которые либо полиномиально либо существенно удовлетворяют сверхтождествам ассоциативности функционального ранга выше 1. Эта характеризация вместе с работами [9], [10] и [12] полностью покрывает все случаи сверхтождеств ассоциативности в полугруппах.

Мы также исследовали необходимые и достаточные условия для полугрупп, которые существенно удовлетворяют сверхтождествам дистрибутивности для всех возможных функциональных рангов. Таким образом, эта характеризация вместе с работой [36] также полностью покрывает все случаи сверхтождеств дистрибутивности в полугруппах.

Далее, мы сосредоточились на интересном расшерении класса сверхассоциативных полугрупп, а именно класса сверхалтернативных полугрупп. Классическая теорема Артина для альтернативных алгебр переносится и доказывается на сверхальтернативные полугруппы, утверждющий, что любые 2 элемента сверхалтернативной полугруппы порождают сверхассоциативную подполугруппу. Вследствие этого, мы также получили необходимые и достаточные условия для полугрупп, которые полиномиально удовлетворяют сверхтождествам альтернативности.

В заключение, мы исследовали необходимые и достаточные условия для полугрупп, которые полиномиально удовлетворяют сверхтождествам транзитивности для все-возможных функциональных рангов. Аналогичные результаты справедливы также в случае существенной выпольнимости сверхтождеств транзитивности в полугруппах. Таким образом, эти характеризации также полностью исчерпают все случаи сверхтождеств транзитивности в полугруппах.