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Ա.04.02-<<Տեսական Ֆիզիկա>> մասնագիտությամբ
ֆիզիկամաթեմատիկական գիտությունների դոկտորի գիտական
աստիճանի հայցման ատենախոսության

ՍԵՂՄԱԳԻՐ

ԵՐԵՎԱՆ – 2016

A. I. ALIKHANIAN NATIONAL SCIENCE LABORATORY

Sarkissian Gor

TWO-DIMENSIONAL CONFORMAL FIELD THEORIES WITH DEFECTS AND BOUNDARIES

SYNOPSIS

of Dissertation in 01.04.02-Theoretical Physics presented for the
degree of doctor in physical and mathematical sciences

YEREVAN-2016

Ատենախոսության թեման հաստատվել է Երևանի Պետական Համալսարանի գիտական խորհրդի կողմից:

Պաշտոնական ընդդիմախոսներ՝ Ֆ.մ. գ.դ., ՌԳԱ թղթ. անդամ
Ա.Բելավին (Լանդաուի անվան ՏՖԻ, Չեռնոգոլովկա,ՌԴ)
Ֆ.մ.գ.դ. Գ.Ջորջաձե (ԹՊՀ, Թբիլիսի, Վրաստան)
Ֆ.մ.գ դ. Ռ.Մանվելյան (ԵրՖԻ)

Առաջատար կազմակերպություն՝ Ռակահի անվան Ֆիզիկայի Ինստիտուտ, Երուսաղեմի Հրեական, Համալսարան, Երուսաղեմ, Իսրայել

Պաշտպանությունը կայանալու է 2016 թ. դեկտեմբերի 20-ին ժամը 14-ին, ԱԱԳԼ-ում գործող ԲՈՀ-ի 024 <<Միջուկի և տարրական մասնիկների ֆիզիկա>> մասնագիտական խորհրդում (Երևան-36, Ալիխանյան եղբ. փ. 2)
Ատենախոսությանը կարելի է ծանոթանալ ԱԱԳԼ գրադարանում :
Սեղմագիրը առաքված է 2016 նոյեմբերի 18-ին:
Մասնագիտական խորհրդի գիտական քարտուղար՝

Ֆիզմաթ. գիտ. դոկտոր Դ. Ռ. Կարախանյան

The subject of the dissertation is approved by the scientific council of the Yerevan State University

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Defense takes place on 20th of December 2016 at 14:00 on meeting of the special council of the Higher Attestation Commission of RA 024 “Nuclear and elementary particles physics”, acting in A.I. Alikhanian National Science Laboratory (Yerevan-36, Brothers Alikhanian 2).
The Dissertation can be learnt at the ANSL library.
The synopsis is sent out on 18th November 2016 .
Scientific secretary of the council d.ph.math.sci Karakhanian D.R.

Abstract

This work is devoted to study of D-branes and defects in two-dimensional conformal field theory.

We study D-branes and defects in WZW and gauged WZW models. We constructed geometrical realization of the non-maximally symmetric parafermionic D-branes in WZW model. Non-maximally symmetric non-factorizable D-branes in product of WZW models are found as well.

We construct geometrical realization of the Cardy D-branes, permutation D-branes and defects in coset models. We find also D-branes in asymmetrically gauged Nappi-Witten cosmological models and in Guadagnini-Martellini-Mintchev model.

We studied in detail the symplectic phase spaces of the WZW and gauged WZW models with defects and established their symplectomorphism with certain symplectic phase spaces of three-dimensional (double) Chern-Simons theory with Wilson lines.

We study D-branes also in Gepner model. In particular we study Cardy D-branes in (2,2,2,2) Gepner model.

We consider topological defects gluing various duality related backgrounds. We constructed duality defects implementing abelian, non-abelian and fermionic T-duality transformations and shown that they are given by the Poincare bundle and its super and non-abelian generalizations. In particular we consider in detail defects implementing T-duality between SU(2) WZW model and lens space, and between axially and vectorially gauged WZW models. We found Fourier-Mukai transform of the Ramond-Ramond fields under non-abelian T-duality generated by the gauge invariant flux of the non-abelian Poincare bundle.

We constructed also topological defects in the Liouville and Toda field theories.

Timeliness and relevance

The study of two dimensional conformal invariant quantum field theory passed long way. Applications to important problems in the various topics of physics are so numerous that conformal field theory became one of the most powerful technique in modern physics. The first great success was the exact computation of critical exponents for the second order phase transition in two dimensional statistical models.

The vast branch of applications of conformal field theory is String theory. String theory offers the most developed candidate for fundamental theory of quantum gravity and approach to the unification of known interactions. Conformal field theories appear as solutions of string equations of motion.

The study of the boundary conditions is very important question in physics. Realistic systems possess boundaries and hence their full understanding requires control of boundary conditions. For two dimensional conformal field theories, study of boundaries was started by John Cardy in sequence of papers, in particular [1,2]. The presence of powerful infinite dimensional symmetries has given rise to vast number exact results on boundary critical exponents and correlation functions.

Boundary conformal field theories are more directly applicable to real physical systems than conformal field theories on closed surfaces. Many processes in three space dimensions have rotational symmetry and all the relevant quantities depend on the time and radial coordinates. Thus, quantum field theories on the half plane appear naturally. Quantum impurity scattering, Kondo effect, is the most prominent example [3].

In String theory, we need two-dimensional conformal field theories to describe open strings.

At the low energy limit, p-branes appear as the supergravity solitons, which describe stable objects whose masses are distributed along p+1-dimensional hypersurfaces in spacetime. Beyond the low energy regime, the supergravity needs to be replaced by the full-fledged String theory, and we need to understand how to describe branes in the String theory. For the large class of branes, which became known as D-branes, the answer was given by Polchinski in [4]: D-branes are objects on which open string can end. The “D” in D-branes stands for the Dirichlet boundary conditions, which constrain the open string endpoints to stay within the brane world-volume.

The importance of D-branes for the understanding of the String theory, and perhaps many other branches of the modern theoretical physics, is enormous. Non-complete list of the applications includes: Brane modelling of gauge theories [5], Braneworld scenario [6,7], Braneworld cosmology and inflation [8], counting of states on the black holes by the superstring theory [9], holographic principle: gravitational description of the quarks, known as the AdS/CFT correspondence [10].

The boundary conformal field theories can be generalized to consider a situation in which two (or more) non-trivial conformal field theories are glued together along a common interface.
Interfaces in two-dimensional theories are oriented lines separating two different quantum field theories. In this dissertation we consider special class of interfaces, for which the energy-momentum tensor is continuous across the defect. These interfaces are called topological defects [11].

During the last years topological defects in two-dimensional quantum field theories appeared in the various topics. Let us mention some of them. Topological defects appear in quantum Hall problem [12], quantum wires problem [13], in the consideration of impurities [14-16]. Topological defects played an important role in the topologically twisted N=4 SYM approach to the geometric Langland program [17]. Defects provide us with examples of 2-category in physics [18-20]. Defects in the Liouville and Toda field theories appear as holographic counterpart of the Wilson lines in the AGT correspondence [21-25]. Defects appear as domain walls in the Ads/CFT correspondence in the presence of D-branes [26]. Recently they were found to be useful also in study of the renormgroup flow [27-28].

The topological defects have proved to be very useful in study of the boundary state transformation. Since the topological defect can be moved to the boundary without changing the correlator, it can be fused with the boundary producing new boundary condition. Remembering that in String theory boundary states correspond to D-branes, one arrives to the conclusion that topological defects induce D-brane transformation. This property was crucial for example in the topologically twisted N=4 SYM approach to the Langland problem [17]. On the other side D-branes are classified by their Ramond-Ramond or K-theory charges. Therefore topological defects should induce also transformations in the cohomology and K-theory groups [29-31].

Aim of the dissertation

1. To study non-maximally symmetric D-branes in the Wess-Zumino-Witten (WZW) models and product of the WZW models.
2. To study D-branes and defects in vectorially gauged WZW model.
3. To construct D-branes in asymmetrically gauged WZW model.
4. To quantize WZW model with defects.
5. To quantize gauged WZW model with defects.
6. To construct topological defects in the Liouville and Toda field theories.
7. To find geometrical realization of the Cardy states in (2,2,2,2) Gepner model.
8. To construct defect implementing various string dualities, namely abelian T-duality, non-abelian T-duality, fermionic T-duality.
9. To find Fourier-Mukai transform for the non-abelian T-duality.

Novelty of the work

In this work the following new result were obtained:

1. Geometric realizations of the Cardy states, permutation branes and defects in coset models were obtained. The geometric meaning of the field identification and selection rules in coset theories was revealed.
2. D-branes corresponding to the non-maximally symmetric Maldacena-Moore-Seiberg states were constructed.
3. New non-factorizable non-maximally symmetric D-branes on a product of WZW models were constructed. It was shown that some of them provide D-branes in certain asymmetrically gauged WZW models.
4. Canonical quantization of WZW and gauged WZW models with D-branes and defects was performed.
5. Topological defects in the Liouville and Toda field theories were constructed.
6. Topological defects implementing various T-dualities were constructed.
7. New approach to the calculation of the Ramond-Ramond fields under the non-abelian T-duality was developed.
8. Geometrical realization some of the Cardy states in the (2,2,2,2) Gepner model was found.

Practical value

The results of this dissertation can have applications to condensed matter problems as well as to String theory. As we mentioned before defects have numerous applications in the condensed matter problems. In particular topological defects play important role also in the recently much discussed entropy entanglement problem.

Our results can be useful in study of the String dualities, which played crucial role almost in all recent works in String theory. As it is well known dualities can be used to derive new background from the existing ones. In particular recently non-abelian T-duality was used to derive new AdS type background. Our findings concerning defects implementing non-abelian T-duality can be used to generalize this program to the superisometry non-abelian T-duality.

Main points to defend

1. We have shown that D-branes in coset models are geometrically realized as a pointwise product of conjugacy classes.
2. We have shown that non-maximally symmetric parafermionic D-branes have geometry of pointwise product of conjugacy class and a U(1) subgroup.
3. We have shown that certain diagonal embedding of the parafermionic branes in a product of WZW models provides D-branes in asymmetrically gauged WZW models like Nappi-Witten and Guadagnini-Martellini-Mintchev models.
4. We established symplectomorphisms between the phase spaces of the WZW and gauged WZW models with defects and branes and phase spaces of the 3D Chern-Simons or double Chern-Simons theory with time-like Wilson lines on a manifold having the form of the product of the certain two-dimensional Riemann surface and the time axis.
5. We found that defects implementing abelian T-duality, non-abelian T-duality and fermionic T-duality are given by the various cousins of the Poincare bundle.
6. In particular we studied in much detail defects implementing abelian T-duality between SU(2) WZW model and lens space, and between axially and vectorially gauged WZW models.

7. We have shown that the transformation of the Ramond-Ramond fields under the non-abelian T-duality can be written as the Fourier-Mukai transform with a kernel given by the flux of the corresponding defect.
8. We found that topological defects in the Liouville and Toda field theories are labeled by the physical and (semi-) degenerate primaries and thus constitute discrete and continuous families.

Approbation of the work

The works on which this dissertation is based on are reported in the following **international conferences:**

1. Workshop Generalized Geometry and T-dualities: May 9-13, 2016, Simons Center for Geometry and Physics, Stony Brook University, Stony Brook, NY, USA.
2. Workshop and School "Selected Topics in Theoretical High Energy Physics", 21-25 September 2015, Tbilisi.
3. Conference on Recent Progress in Quantum Field Theory and String Theory, (smr 2773), 14-19 September 2015, Yerevan.
4. IX International Symposium on Quantum Theory and Symmetries (QTS-9), Yerevan, Armenia, July 13-18, 2015.
5. Second Autumn School on High Energy Physics and Quantum Field Theory, Yerevan, 6-10 October, 2014.
6. Frontiers in field and string theory, Yerevan Physics Institute, Yerevan 22-26, September, 2014.
7. The International Workshop "Supersymmetry in Integrable Systems", Yerevan, 27-30 August, 2012.
8. The International Workshop "Supersymmetry in Integrable Systems", Yerevan, 24-28 August 2010.

9. Workshop Geomaps Retreat, Spring 2010, University of Aarhus, Sandbjerg Estate, Denmark.
10. Workshop CTQM Nielsen Retreat , 2009, University of Aarhus, Sandbjerg Estate, Denmark.

and **invited seminar talks** in

1. L.D.Landau Institute for Theoretical Physics, Chernogolovka, Russia, 2016.
2. University of Rome “Tor Vergata”, Rome, Italy 2014.
3. Arnold Sommerfeld Center, Ludwig Maximilian University, Munich, Germany 2014.
4. The Abdus Salam ICTP, Trieste, Italy 2013.
5. The Racah Institute of Physics, The Hebrew University of Jerusalem, Israel 2013.
6. Tel Aviv University, TelAviv, Israel, 2012.
7. Weizmann Institute of Science, Rehovot, Israel 2012.
8. King’s College, London, UK, 2008.
9. Ecole Polytechnique, Paris, France 2003.

and also on numerous seminars of the chair of theoretical physics of Yerevan State University and division of theoretical physics of Yerevan Physics Institute.

Publications

The dissertation is based on 20 **published** works listed at the end.

Length and structure of the dissertation

The dissertation contains an Introduction, 8 chapters, the last chapter contains conclusions, 5 appendices, list of bibliography with 193 names and has 289 pages.

Content of the dissertation

The dissertation consists of 8 chapters. In chapter 1 we review the material necessary to present our findings. In chapters 2-7 we deliver our findings. The last chapter 8 contains the list of the findings.

In chapter 1 we collect and review the necessary stuff and technique of two-dimensional conformal field theory. In *section 1.1* we review two-dimensional conformal field theory on closed surfaces. *In section 1.2* we collect all the necessary gadgets of conformal field theory on a world-sheet with a boundary. *In section 1.3* we review topological defects. *In section 1.4* we illustrate the developed technique in important case of free boson theory.

In section 1.5 we introduce Wess-Zumino-Witten (WZW) model and gauged WZW model (coset models).

In chapter 2 we study non-maximally symmetric branes on WZW model, preserving only part of the diagonal affine symmetry. *In section 2.1* we analyze properties of WZW model on a world-sheet with a boundary. *In section 2.2* we study so called parafermionic D-branes. *In subsection 2.2.1* we define non-maximally symmetric D-branes, sometimes called also parafermionic, as pointwise product of the conjugacy classes and a U(1) subgroup. We construct Lagrangian with the boundary conditions constraining group field to take on a boundary values in the parafermionic D-brane world-volume.

We study symmetries of the action and show that it is invariant under axial combination of the left and right U(1) currents, and vectorial combination of the currents belonging to the subgroup commuting with U(1) group. *In subsection 2.2.2* we study geometry of the parafermionic D-branes for SU(2) group and show that generically it is three-dimensional and given by an inequality constraining the values of the second Euler angle. *In subsection 2.2.3* we review construction of the boundary states of the parafermionic D-branes for SU(2) group, called MMS (Maldacena-Moore-Seiberg) states, given in [32]. *In subsection 2.2.4* we compute the overlap of the MMS boundary state with the graviton wave packet and show that in the semiclassical limit it gives the inequality derived in subsection 2.2.2. *In section 2.3* we study permutation branes on a K+1-fold product of group G on a world-sheet with a boundary, with boundary condition constraining product of group fields to take value again in discrete set of conjugacy classes defined in 2.1.2. *In subsection 2.3.1* we describe geometry of the permutation branes. *In subsection 2.3.2* we write the Lagrangian with these boundary conditions and show that it has symmetries of permutation branes studied in 1.2.4. *In subsection 2.3.3* we compute for SU(2) group overlap of the permutation boundary states defined in 1.2.4 with the graviton wave packet and show that in the semiclassical limit they indeed have the geometry described in 2.3.1. *In section 2.4* we construct type I non-maximally symmetric non-factorizable branes on a product of identical groups. *In subsection 2.4.1* we define new branes as product of permutation branes studied in 2.3 with elements of an U(1) subgroup. We construct Lagrangian with these boundary conditions and study their symmetries. *In subsection 2.4.2* we study geometry of these branes for SU(2) x SU(2) group. *In subsection 2.4.3* we construct boundary states of the type I branes for SU(2) x SU(2) group, compute the overlap with the graviton wave packet and show that it is in agreement with the calculations in 2.4.2. We also check that type I boundary states satisfy the Cardy criteria. *In section 2.5* we study type II non-maximally symmetric non-factorizable branes on a product of identical groups. *In subsection 2.5.1* we define new branes as product of permutation branes studied in 2.3 with elements of two U(1) subgroups. We construct Lagrangian with these boundary conditions and study their symmetries. *In subsection 2.5.2* we study geometry of these branes for SU(2) x SU(2) group. *In subsection 2.5.3* we construct boundary states of the type II branes for SU(2) x SU(2) group, compute the overlap with the graviton wave packet and show that it is in agreement with the calculations in 2.5.2 We also check that type II boundary states satisfy the Cardy criteria.

In chapter 3 branes and defects in gauged WZW model are constructed. *In section 3.1* we study branes in the vectorially gauged WZW model G/H. *In subsection 3.1.1* we construct D-branes in the vectorially gauged WZW model using the representation of the gauged WZW model Lagrangian via the auxiliary fields reviewed in 1.5.5. and the action of the WZW model with a boundary presented in 2.1.1. Analysing global issues mentioned in 2.1.2 we find correspondence of the found D-branes with the Cardy states of a coset model in the absence of the common center of G and H. *In subsection 3.1.2* we analyze special case of a coset when G and H have common center. We show that found D-branes satisfy present in this case field identification and selection rules for the primary fields of coset models. *In section 3.2* we present the Lagrangian approach to defects in WZW models. *In section 3.3* we construct Cardy defects and permutation branes in vectorially gauged WZW model. *In subsection 3.3.1*, using again the reoresentation of the gauged WZW model Lagrangian via the auxiliary fields presented in 1.5.5 and the Lagrangian of the WZW model with defects reviewed in 3.2., the geometry and action of the topological defects and permutation branes in GWZW are constructed. We show that they are in one-to-one correspondence with the primary fields of coset models. *In subsection 3.3.2* we consider overlap of the permutation brane boundary state on product of cosets SU(2)(k)/U(1) x SU(2)(k)/U(1) with the graviton wave packet and show that it has the geometry found in 3.3.1. *In section 3.4* we consider D-branes in asymmetrically gauged cosmological Nappi-Witten model and in the Guadagnini-Martellini-Mintchev mode. *In subsection 3.4.1* we present D-branes in the Nappi-Witten model, construct the action with these boundary conditions and check gauge invariance. *In subsection 3.4.2* we study in detail D-branes in the Nappi-Witten cosmological model SL(2,R) x SU(2)/U(1) x U(1) and present the explicit equations of the corresponding D-brane hypersurface. *In subsection 3.4.3*, in a similar way D-branes in the Guadagnini-Martellini-Mintchev model are considered. *In subsection 3.4.4* we consider in detail D-branes in the SU(2) x SU(2)/U(1) GMM model. We show that D-branes are classified according to the Cardy theorem.

In chapter 4 we establish symplectomorphisms between certain phase spaces of the three-dimensional Chern-Simons and double Chern-Simons theories and that of WZW and gauged WZW models with branes and defects. *In section 4.1* we review three-dimensional Chern-Simons theory with sources on a product of a Riemann surface and a time line. *In subsection 4.1.1* we elaborate phase space of the Chern-Simons theory and show that it is given by the moduli space of flat connections on the Riemann surface punctured at the points where Wilson line hit it, with the holonomies around punctures belonging to the set of conjugacy classes considered in 2.1.2. *In subsection 4.1.2* we present details on symplectic forms on moduli space of flat connections on two-dimensional sphere with Wilson lines and holes. *In subsection 4.1.3* we present symplectic forms on moduli space of flat connections on a Riemann surface of genus g with Wilson lines. *In subsection 4.1.4* we write down the symplectic form of the double Chern-Simons theory. *In section 4.2* we establish symplectpmorphisms between certain phase space of the Chern-Simons theory and that of WZW model with branes and defects. *In subsection 4.2.1* we compare the Hilbert space of the Chern-Simons theory with Wilson lines on certain spaces and that of WZW models with branes and defects and list the statements which we prove here. *In subsection 4.2.2* we review bulk WZW model and establish that the symplectic phase space of the WZW model on circle coincide with that of CS theory on annulus [33]. *In subsection 4.2.3* we recall that the symplectic phase space of the WZW model on a strip coincide with that of CS theory on a disc with two Wilson lines [34]. *In subsection 4.2.4* we establish that the symplectic phase space of the WZW model with a defect is symplectomorphic to that of CS theory on an annulus with a Wilson line. *In subsection 4.2.5* we establish that the symplectic phase space of the WZW model on a strip with a defect inserted is symplectomorphic to that of CS theory ona disc with three Wilson lines. *In subsection 4.2.6* we establish that the symplectic phse space of the WZW model G x G on a strip with boundary conditions specified by permutation branes coincide with that of CS on an annulus with two Wilson lines. *In section 4.3* we perform canonical quantization of the vectorially gauged WZW model G/H with defects and boundaries and establish symplectomorphisms between their phase spaces and certain phase spaces of the double Chern-Simons theories. *In* *subsection 4.3.1* we present short summary of the statements proved in this section. *In subsection 4.3.2* we review bulk gauged WZW model and show that its phase space on a cylinder coincides with that of double CS theory [35,36] on product of an annulus and time-line. *In subsection 4.3.3* we show that the phase space of the gauged WZW model on a cylinder with a defect line coincides with that of double CS theory on a product of an annulus and time-line with gauge fields of groups G and H coupled to a Wilson line. *In subsection 4.3.4* we show that the phase space of the gauged WZW model on a strip with a defect line coincides with that of the double CS theory on a disc times time-line with gauge field of groups G and H coupled to three Wilson lines. *In section 4.4* we analyze especially interesting case of the topological coset G/G. *In subsection 4.4.1* we analyze bulk G/G coset and show that the phase space of a bulk G/G theory on a cylinder is symplectomorphic to that of a CS theory on a product of two-torus and time-line. *In subsection 4.4.2* we show that the topological coset G/G on a cylinder with a defect line is isomorphic with that of a CS theory on a product of two-torus and time-line with two Wilson lines. *In subsection 4.4.3* we demonstrate the symplectomorphism of the phase space of G/G topological coset on a strip with a defect line and a CS theory on product of two-sphere and time-line with six Wilson lines. *In section 4.5* we analyze a product of cosets G/H x G/H on a strip with boundary conditions specified by permutation branes and show that its phase space is symplectomorphic to the phase space of the double CS theory on an annulus times the time-line and with G and H gauge fields both coupled to two Wilson lines. *In section 4.6* we establish symplectomorphism of the phase space of product topological cosets G/G x G/G on a strip with boundary conditions given by the permutation branes and that of CS theory on a torus times the time-line with four Wilson lines.

In chapter 5 we study topological defects implementing various dualities. In section 5.1 we review some basic facts concerning topological defects and their relation to T-duality. It is established that the defect implementing bosonic T-duality is given by the Poincare line bundle. We demonstrated in the simple example of a scalar field compactified on a circle how the defect equations of motion reproduce the appropriate duality transformations. *In section 5.2* we generalize this to the factorized T-duality in non-linear sigma models with isometries. We also present a defect generating a combined action of the Z(k) orbifolding together with a T-duality transformation. *In section 5.3* we explain how the T-duality transformation of the Ramond-Ramond charges can be written as the Fourier-Mukai transform with the kernel given by the exponent of the gauge invariant flux on the corresponding topological defect. *In section 5.4* we study T-dualities in the special case of the SU(2) WZW model and a lens space. *In subsection 5.4.1* we review [37] kernel of the Fourier-Mukai transform of the T-duality between SU(2) WZW model and lens space implementing the map between the corresponding twisted cohomology group. *In subsection 5.4.2* we use the fact that defects (at least in rational conformal field theories) are uniquely determined y their action on bulk fields. We construct several families of defects by using T-duality and orbifolding. *In subsection 5.4.3* for one such family we determine the geometry of the underlying bibranes. We recover structure familiar from the Fourier-Mukai transform studied in 5.4.1. *In section 5.5* we construct defect between axial and vector gauging of G/U(1) gauged WZW models for a general group G. For the case of G=SU(2) the geometrical construction is translated to the algebraic parafermionic language. *In subsection 5.5.1* we present geometry and flux of the defect gluing axially-vectorially gauged models. *In subsection 5.5.2* we specialize to group SU(2) and show that for level k parafermions there are k+1 topological defects mapping axially gauged SU(2)/U(1) cosets to the vectorially gauged SU(2)/U(1) coset, labeled by the integrable spin j=0,…k/2. *In subsection 5.5.3* we construct them as the appropriate operators in the parafermion Hilbert space. We show that the defect corresponding to j=0 implements Z(k) orbifolding together with T-duality. These defects project A(j,n) Cardy branes in SU(2)/U(1) coset to the B(j) branes constructed in [32]. *In section 5.6* we study the defects performing the fermionic T-duality [38]. *In subsection 5.6.1* we review the necessary information on pseudodifferential forms integration. *In subsection 5.6.2* we review the fermionic T-duality. *In subsection 5.6.3* we show that the defect inducing the fermionic T-duality is given by the fermionic generalization of the Poincare line bundle, which we call Super-Poincare line bundle. We demonstrate that the defect equations of motion reproduce the fermionic T-duality transformation rules found in. *In subsection 5.6.4* using the exponent of the gauge invariant flux on this defect as a kernel of the Fourier-Mukai transform with a pushforward map given by the fiberwise integration on supermanifold, we derive the transformation of the Ramond-Ramond fields under the fermionic T-duality. *In section 5.7* we construct topological defects producing non-abelian T-duality. *In subsection 5.7.1* we review non-abelian T-duality. In particular we recall the duality relations and demonstrate general formulas for the case of SU(2) principal chiral model. *In subsection 5.7.2* we present defect performing non-abelian T-duality and show that the defect equations of motion reproduce the duality relations derived in subsection 5.7.1. *In subsection 5.7.3* using the flux of non-abelian T-duality defect derived in subsection 5.7.2 we derive the Fourier-Mukai transform formula for non-abelian T-duality and compute the Ramond-Ramond fields transformation for SU(2) isometry group. We obtain that our results are in agreement with that of [39,40].

In chapter 6 we study topological defects in the Liouville and Toda field theories. *In section 6.1* we write down topological defects in the Liouville field theory. It is shown that defects are labeled by the physical and degenerate primaries of Liouville field theory, and correspondingly compose discrete and continuous families. We also have shown that the well known relation [41] between the OPE structure constants and the fusing matrix with an intermediate entry set to the vacuum proved for rational conformal field theories, holds also for the Liouville field theory. *In section 6.2* we write down topological defects in the Toda field theory. We have shown that topological defects in Toda field theory are labeled by the physical, semi-degenerate and fully degenerate primaries. *In section 6.3* we analyze classical Liouville field theory with defects. *In subsection 6.3.1* we review the general solution of the Liouville equations. *In subsection 6.3.2* we present general solution of the defect equations of motion. *In section 6.4* we review the heavy asymptotic semiclassical limit. *In section 6.5* we calculate the defect two-point function in the heavy asymptotic limit. *In subsection 6.5.1* we calculate heavy asymptotic limit of defect two-point functions. *In subsection 6.5.2* we show that the heavy asymptotic limit of defect two-point function found in the previous section is given by exponential of the action with defects evaluated on solution of defect equations of motion with two singularities.

In chapter 7 we study Cardy states in (2,2,2,2) Gepner model. *In section 7.1* we review necessary background material on the simple current extensions. *In section 7.2* we review Gepner

model via simple current extension formalism. *In section 7.3* we write down all the necessary information on the (2,2,2,2) Gepner model: orbit representatives, conformal weights. Using the resolved characters we compute the torus partition function and show that it coincides with the one computed in the appendix 5 as an orbifold partition function at the SU(2) x SU(2) x SU(2) x SU(2) point. Using the general formulae of section 7.1 we also derive the annulus partition functions between the various Cardy states, paying special attention to the peculiarities caused by the presence of the fixed points. *In section 7.4* we study D0-branes on the orbifold T4/Z4. We compute all the annulus [artition functions between D0-branes located at points in T4/Z4 orbifold that are fully or partially fixed points under the orbifold group action. Using previously derived formulae for the annulus partition functions between Cardy states of the (2,2,2,2) model we establish a partial dictionary between Cardy states and D0-branes.
In chapter 8 we presented list of our main findings.

In five appendices some technical points are collected. *In appendix 1* double Gamma and Sinus functions are reviewed. *In appendix 2* asymptotic behavior of Gamma function is reviewed. *In appendices 3 and 4* some identities on Theta functions are collected. *In appendix 5* some technical points on calculation of the partition function of T4/Z4 orbifold are delivered.

Conclusions

Let us briefly summarize our findings.

1. We constructed geometrical realization of the Cardy states in coset model. We found geometrical meaning of the field identification and selection rules for primary fields of coset model.
2. We found geometrical realization of the parafermionic D-branes in WZW model.
3. We constructed non-maximally symmetric non-factorizable D-branes in product of WZW models.
4. We have found geometrical realization of permutation branes and defects in coset models.
5. We have shown that certain diagonal embedding of the parafermionic D-branes in product of WZW models provide D-branes in the Nappi-Witten cosmological model as well as in the Guadagnini-Martellini-Mintchev model.
6. We proved symplectomorphism between phase space of the WZW nodel with boundaries and defects and that of 3D Chern-Simons theory with Wilson lines on a product of a Riemann surface and time-line.
7. We proved symplectomorphism between phase space of the gauged WZW model with boundaries and defects and that of 3D double Chern-Simons theory on product of a Riemann surface and time-line.
8. We constructed topological defects implementing abelian, non-abelian and fermionic T-dualities. We have shown that they are given by the Poincare bundle and its non-abelian and super generalizations correspondingly.

9. We studied in detail defects implementing T-duality between SU(2) WZW model and lens space. We have paid also special attention to the Fourier-Mukai transform of the twisted cohomology groups generated by the gauge invariant flux of this defect.
10. We studied in detail defects implementing T-duality between axially and vectorially gauged WZW model.
11. We calculated Fourier-Mukai transform of the Ramond-Ramond fields under non-abelian T-duality.
12. We have shown that fusion matrix of the Liouville field theory with the intermediate state set to the vacuum gives rise to the DOZZ structure constants.
13. We constructed topological defects in the Liouville and Toda field theories as intertwining operators using Cardy-Lewellen cluster equation. We have shown that in the Liouville field theory defects are labeled by the physical and degenerate primaries. We proved that in Toda field theory topological defects are labeled by the physical, semi-degenerate and degenerate primaries.
14. We studied Lagrangian of the Liouville theory with defects and shown its agreement with the operator description in the semiclassical limit.
15. We found geometrical realization of the Cardy states in (2,2,2,2) Gepner model, using its equivalence with T4/Z4 ordfold.

References

1. J. L. Cardy, "Conformal Invariance and Surface Critical Behaviour", Nucl. Phys. B **240** (1984) 514.
2. J. L. Cardy, "Boundary Conditions, Fusion Rules and the Verlinde Formula", Nucl. Phys. **324** (1989) 581.
3. I. Affleck, "Conformal field theory approach to the Kondo effect", Acta Phys. Polon. B **26** (1995) 1869.
4. J. Polchinski, "Dirichlet Branes and Ramond-Ramond charges," Phys. Rev. Lett. **75** (1995) 4724.
5. A. Giveon and D. Kutasov, "Brane dynamics and gauge theory," Rev. Mod. Phys. **71** (1999) 983.
6. N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "The Hierarchy problem and new dimensions at a millimeter", Phys. Lett. B **429** (1998) 263.
7. L. Randall and R. Sundrum, "A Large mass hierarchy from a small extra dimension", Phys. Rev. Lett. **83** (1999) 3370.
8. G. R. Dvali and S. H. H. Tye, "Brane inflation", Phys. Lett. B **450** (1999) 72.
9. A. Strominger and C. Vafa, "Microscopic origin of the Bekenstein-Hawking entropy," Phys. Lett. B **379** (1996) 99
10. O. Aharony, S. S. Gubser, J. M. Maldacena, H. Oogury and Y. Oz, "Large N field theories, string theory and gravity," Phys. Rept. **323** (2000) 183.
11. C. Bachas and M. Gaberdiel, "Loop operators and the Kondo problem," JHEP **0411** (2004) 965.
12. P. Fendley, M. P. A. Fisher and C. Nayak, "Boundary Conformal field Theory and Tunneling of Edge Quasiparticles in non-abelian Topological States", **324** (2009) 1547.
13. E. Wang and I. Affleck, "Tunneling in quantum wires: A Boundary conformal field theory approach," Nucl. Phys. B **417** (1994) 403.

14. M. Oshikawa and I. Affleck, “Boundary conformal field theory approach to the critical two-dimensional Ising model with a defect line”, Nucl. Phys. B **495** (1997) 533.
15. H. Saleur, “Lectures on nonperturbative field theory and quantum impurity problems,” cond-mat/9812110.
16. H. Saleur, “Lectures on nonperturbative field theory and quantum impurity problems: Part 2,” cond-mat/0007309.
17. A. Kapustin and E. Witten, “Electric-Magnetic Duality And The Geometric Langlands program,” Commun. Num. Phys. **1** (2007) 1.
18. A. Davydov, L. Kong and I. Runkel, “Invertible defects and isomorphisms of rational CFTs”, Adv. Theor. Math. Phys. **15** (2011) no.1, 43.
19. J. Frohlich, J. Fuchs, I. Runkel and C. Schweigert, “Duality and defects in rational conformal field theory,” Nucl. Phys. B **763** (2007) 354.
20. C. Schweigert, J. Fuchs and I. Runkel, “Categorification and correlation functions in conformal field theory,” in: Proceeding of the ICM, 2006, Madrid, M. Sanz-Sole, J. Soria, J. L. Varona, eds European Mathematical Society, Zurich (2006) 443.
21. N. Drukker, D. Gaiotto and J. Gomis, “The Virtue of Defects in 4D Gauge Theories in 2D CFT’s”, JHEP **1106** (2011) 025.
22. L. F. Alday, D. Gaiotto and Y. Tachikawa, “Liouville Correlation Functions from Four-dimensional Gauge Theories”, Lett. Math. Phys. **91** (2010) 167.
23. L.F. Alday, D. Gaiotto, S. Gukov, Y. Tachikawa and H. Verlinde, “Loop and surface operators in N=2 gauge theory and Liouville modular geometry,” JHEP **1001** (2010) 113.
24. N. Drukker, J. Gomis, T. Okuda and J. Teschner, “Gauge Theory Loop Operators and Liouville Theory”, JHEP **1002** (2010) 057.
25. V. B. Petkova, “On crossing relations in the presence of defects”, JHEP **1004** (2010) 061.
26. C. Bachas, J. de Boer, R. Dijkgraaf and H. Ooguri, “Permeable conformal walls and holography”, JHEP **0206** (2002) 027.

27. M. Kormos, I. Runkel and G. M. T. Watts, "Defects flows in minimal models", JHEP **0911** (2009) 057.
28. D. Gaiotto, "Domain Walls for Two-dimensional Renormalization Group Flows," JHEP **1212** (2012) 103.
29. J. Fuchs, C. Schweigert and K. Waldorf, "Bi-branes: Target space geometry for world-sheet topological defects," J. Geom. Phys. **58** (2008) 576.
30. I. Brunner, H. Jockers and D. Roggenkamp, "Defects and D-Brane Monodromies," Adv. Theor. Math. Phys. **13** (2009) 1077.
31. C. Bachas, I. Brunner and D. Roggenkamp, "A world-sheet extension of O(d,d,Z)", JHEP **1210** (2012) 039.
32. J. M. Maldacena, G. W. Moore and N. Seiberg, "Geometrical interpretation of D-branes in gauged WZW models," JHEP **0107** (2001) 046.
33. S. Elitzur, G. W. Moore, A. Schwimmer and N Seiberg, "Remarks on the Canonical Quantization of the Chern-Simons-Witten Theory," Nucl. Phys. B **326** (1989) 108.
34. K. Gawedzki, I. Todorov and P. Tran-Ngoc-Bich, "Canonical quantization of the boundary Wess-Zumino-Witten model", Commun. Math. Phys. **248** (2004) 217.
35. K. Gawedzki, "Boundary WZW, G/H, G/G and CS theories," Annals Henri Poincare **3** (2002) 847.
36. G. W. Moore and N. Seiberg, "Taming the conformal Zoo", Phys. Lett. B **220** (1989) 422.
37. P. Bouwknegt, J. Evslin and V. Mathai, "T duality: Topology change from H flux", Commun. Math. Phys. **249** (2004) 383.
38. N. Berkovits and J. Maldacena, "Fermionic T-duality, Dual Superconformal Symmetry, and the Amplitude/Wilson Loop Connection", JHEP **0809** (2008) 062.
39. K. Sfetsos and D. C. Thompson, "On non-abelian T-dual geometries with Ramond fluxes", Nucl. Phys. B **846** (2011) 21.

40. G. Itsios, Y. Lozano, E. O. Colgain and K. Sfetsos, "Non-Abelian T-Duality and consistent truncation in type-II supergravity," JHEP **1208** (2012) 132.
41. R. E. Behrend, P. A. Pearce, V. B. Petkova and J. B. Zuber, "Boundary conditions in rational conformal theories," Nucl. Phys. B **570** (2000) 525 [Nucl. Phys. B **579** (2000) 707].

List of publications on which the dissertation is based on

1. S. Elitzur, E. Rabinovici and G. Sarkissian, "On least action D-branes," Nucl. Phys. B **541** (1999) 246.
2. S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici and G. Sarkissian, "D-branes in the background of NS five-branes," JHEP **0008** (2000) 046.
3. S. Elitzur and G. Sarkissian, "D-branes on a gauged WZW model," Nucl. Phys. B **625** (2002) 166.
4. G. Sarkissian, "Non-maximally symmetric D-branes on group manifold in the Lagrangian approach," JHEP **0207** (2002) 033.
5. G. Sarkissian, "On DBI action of the nonmaximally symmetric D-branes on SU(2)," JHEP **0301** (2003) 058.
6. G. Sarkissian, "On D-branes in the Nappi-Witten and GMM models," JHEP **0301** (2001) 059.
7. G. Sarkissian and M. Zamaklar, "Diagonal D-branes in product spaces and their Penrose limits", JHEP **0403** (2004) 005.
8. G. Sarkissian and M. Zamaklar, "Symmetry breaking, permutation D-branes on group manifolds: Boundary states and geometric description," Nucl. Phys. B **696** (2004) 66.
9. G. Sarkissian, "Generalised permutation branes on a product of cosets G(k(1))/H x G(k(2))/H," Nucl. Phys. B **747** (2006) 423.

10. G. Sarkissian, “On Cardy states in the (2,2,2,2) Gepner model,” Nucl. Phys. B **769** (2007) 287.
11. P. Anastasopoulos, M. Bianchi, G. Sarkissian and Y. Stanev, “On gauge couplings and thresholds in Type I Gepner models and otherwise,” JHEP **0703** (2007) 059.
12. G. Sarkissian and c. Schweigert, “Some remarks on defects and T-duality”, Nucl. Phys. B **819** (2009) 478.
13. G. Sarkissian, “Defects and Permutation branes in the Liouville field theory”, Nucl. Phys. B **821** (2009) 607-625.
14. G. Sarkissian, “Canonical quantization of the WZW model with defects and Chern-Simons theory,” Int. J. Mod. Phys. A **25** (2010) 1367.
15. G. Sarkissian, “Defects in G/H coset, G/G topological field theory and discrete Fourier-Mukai transform,” Nucl. Phys. B **846** (2011) 338.
16. G. Sarkissian, “On canonical quantization of the gauged WZW model with permutation branes,” Int. J. Mod. Phys. A **26** (2011) 4647.
17. G. Sarkissian, “Some remarks on D-branes and defects in Liouville and Toda field theories,” Int. J. Mod. Phys. A **27** (2012) 1250181.
18. S. Elitzur, B. Karni, E. Rabinovici and G. Sarkissian, “Defects, Super-Poincare line bundle and Fermionic T-duality,” JHEP **1304** (2013) 088.
19. E. Gevorgyan and G. Sarkissian, “Defects, Non-abelian T-duality, and the Fourier-Mukai transform of the Ramond-Ramond fields,” JHEP **1403** (2014) 035.
20. H. Poghosyan and G. Sarkissian, “On classical and semiclassical properties of the Liouville theory with defects,” JHEP **1511** (2015) 005.

Двумерные конформные теории поля с дефектами и границами

Резюме

В этой работе мы изучаем конформные граничные условия и топологические дефекты в двумерных конформных теориях поля.

Мы изучаем дефекты и D-браны в следующих теориях: модели Весса-Зумино-Виттена (ВЗВ), калибровочные модели Весса-Зумино-Виттена (косет модели), теории Лиувилля и Тода, модель Гепнера. Также мы изучаем дефекты, выполняющие струнные дуальности. Перечислим основные результаты.

1. Показано, что немаксимально-симметричные D-браны в ВЗВ теории, заданные состояниями Малдасены-Мура-Зайберга , геометрически описываются поточечным произведением класса сопряженности и U(1) подгруппы. Эти D-браны также иногда называют парафермионными.
2. Доказано, что поточечное произведение классов сопряженности дает геометрическую реализацию состояний Карди в косет теориях.
3. Построены немаксимально-симметричные нефакторизуемые D- браны в произведении ВЗВ моделей.
4. Найдена геометрическая реализация дефектов и перестановочных D-бран в косет теориях.
5. Показано, что диагональное вложение парафермионных D-бран в произведение групп дает D-браны в ассиметрично калибровочных ВЗВ теориях. В частности найдены D-браны в моделях Наппи-Виттена и Гуаданьнини-Мартелини-Минтчева.
6. Установлены симплектоморфизмы между фазовыми пространствами модели ВЗВ с дефектами с одной стороны и фазовым пространсвом трехмерной калибровочной теории Черна-Саймонса с другои стороны с временоподобными Вильсоновскими линиями на многообразии, имеющем вид произведения некоторой римановой поверхности и временной оси.
7. Установлены симплектоморфизмы между фазовыми пространствами калибровочной модели ВЗВ с дефектами с одной стороны и фазовым пространсвом трехмерной двойной (double) калибровочной теории Черна-Саймонса с

другои стороны с временоподобными Вильсоновскими линиями на многообразии, имеющем вид произведения некоторой римановой поверхности и временной оси.

8. Показано, что матрица слияния в теории Лиувилля с вакуумным промежуточным элементом пропорциональна ДОЗЗ (DOZZ) структурным константам.
9. Построены дефекты в теориях Лиувилля и Тода и показано, что они имеют те же индексы, что и вырожденные, физические и полу-вырожденные первичные поля.
10. Изучен Лагранжиан теории Лиувилля с границами и установлено его соотвествие с операторным подходом в квазиклассическом пределе.
11. Построены дефекты, выполняющие абелеву, неабелеву и фермионную Т-дуальности. Показано, что они описываются расслоением Пуанкаре и его обобщениями.
12. В частности мы подробно изучили дефекты, осуществляющие Т-дуальность между SU(2) ВЗВ моделью и пространством линз. В деталях изучено преобразование Фурье-Мукая, порожденное калибровочно инвариантным потоком этих дефектов на пространстве когомологий.
13. Также подробно изучена Т-дуальность между аксиально и векторно калибровочными ВЗВ моделями.
14. Используя дефекты, осуществляющие неабелеву Т-дуальность, мы нашли новый способ вычисления преобразования антисимметричных полей при преобразовании неабелевой Т-дуальности.
15. Мы нашли геометрическую реализацию некоторых состояний Карди в (2,2,2,2) моделе Гепнера, используя ее эквивалентность T4/Z4 тороидальному орбифолду.

Երկչափ կոնֆորմ դաշտի տեսություններն դեֆեկտների և եզրերի առկայությամբ

Ամփոփում

Այս աշխատանքում մենք ուսումնասիրում ենք կոնֆորմ եզրային պայմանները և դեֆեկետները երկչափանի կոնֆորմ դաշտի տեսություններում:
Ուսումնասիրված են դեֆեկտները և D-բրանները հետևյալ տեսություններում՝ Վես-Զումինո-Վիտենի (ՎԶՎ) մոդելում, տրամաչափական ՎԶՎ մոդելում (քոսեթ տեսությունները), Լիուվիլի և Թոդայի տեսություններում, և Գեպների մոդելում: Մենք նաև ուսումնասիրում ենք դեֆեկտները, որոնք իրականացնում են լարային դուալությունները:
Թվենք հիմնական արդյունքները:

1. Ցույց է տրված որ ոչ-մաքսիմալ սիմետրիկ D-բրանները ՎԶՎ մոդելում, որոնք համապատասխանում են Մալդասենա-Մուռ-Զայբերգի վիճակներին, երկրաչափորեն կարող են նկարագրվել որպես համալուծության դաս բազմապատկված U(1) ենթախմբով: Այդ D-բրանները կոչվում են երբեմն պարաֆերմիոնային:
2. Ապացուցված է, որ համալուծության դասերի արտադրյալը տալիս է Քարդիի վիճակների երկրաչափական պատկերը քոսեթ տեսություններում:
3. Կառուցված են ոչ-մաքսիմալ սիմետրիկ ոչ-ֆակտորիզացվող D-բրաններ ՎԶՎ մոդելների արտադրյալում:
4. Գտնված են դեֆեկտների և տեղափոխությունների (permutation) D-բրանների երկրաչափական պատկերը քոսեթ տեսություններում:
5. Ցույց է տրված որ պարաֆերմիոնային D-բրանների անկյունագծային ներդրումը խմբերի արտադրյալի մեջ տալիս է D-բրաններ ասիմետրիկ տրամաչափական ՎԶՎ տեսություններում: Մասնավորապես, գտնված են D-բրանները Նափի-Վիտենի և Գուադանինի-Մարթելինի-Մինթչնի մոդելներում:
6. Հաստատված է սիմպլեկտոմորֆիզմ, մի կողմից, ՎԶՎ մոդելի դեֆեկտների առկայությամբ ֆազային

տարածության և, մյուս կողմից, որոշակի երեք չափանի Չեռն-Սայմոնսի տրամաչափական տեսության Վիլսոնի գծերի առկայությամբ ֆազային տարածության միջև:

7. Հաստատված է սիմպլեկտոմորֆիզմ, մի կողմից, տրամաչափական ՎՁՎ մոդելի դեֆեկտների առկայությամբ ֆազային տարածության և, մյուս կողմից, որոշակի երեք չափանի կրկնակի Չեռն-Սայմոնսի տրամաչափական տեսության Վիլսոնի գծերի առկայությամբ ֆազային տարածության միջև:
8. Ապացուցված է, որ Լիուվիլի տեսության միաձուլման մատրիցան, որի միջանկյալ էլեմենտներից մեկը վակուումային վիճակն է, համեմատական է ԴՈԶԶ (DOZZ) կառուցվացքային հաստատունին:
9. Կառուցված են դեֆեկտները Լիուվիլի և Թոդայի տեսություններում և ապացուցված է, որ նրանք գտնվում են մեկ-մեկ համապատասխանության մեջ ֆիզիկական, այլասերված և կիսաայլասերված առաջնային դաշտերի հետ:
10. Ուսումնասիրված է Լիուվիլի տեսության Լագրանժիանը՝ դեֆեկտների առկայությամբ և հաստատված է նրա համապատասխանությունը օպերատորային մոտեցման հետ:
11. Կառուցված են դեֆեկտներ իրականացնող աբելյան, ոչ-աբելյան և ֆերմիոնային T-դուալությունները:
12. Մասնավորապես, մանրամասնորեն հետազոտված են դեֆեկտներ իրականացնող աբելյան T-դուալությունը SU(2) ՎՁՎ մոդելի և ոսպնապակի տարածության միջև:
13. Ուսումնասիրված է T-դուալությունը աքսիալ ու վեկտորական տրամաչափական ՎՁՎ մոդելների միջև:
14. Օգտագործելով դեֆեկտներ իրականացնող ոչ-աբելյան T-դուալությունը, մշակված է նոր մեթոդ Ռամոն-Ռամոն դաշտերի ձևափոխությունը հաշվարկելու համար:
15. Օգտագործելով (2,2,2,2) Գեպների մոդելի էկվիվալենտությունը T4/Z4 օրբիֆոլդին, գտնված են որոշ Քարդիի վիճակների երկրաչափական պատկերները: