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ԵՐԵՎԱՆ, 2019

YEREVAN STATE UNIVERSITY

Nerses Vilyam Srapionyan

Greedy algorithms in muldimensional Banach spaces

SYNOPSIS

of the thesis for the degree of candidate of
physical and mathematical sciences in the specialty
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YEREVAN, 2019

Ատենախոսության թեման հաստատվել է Երևանի Պետական Համալսարանում

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	Ս. Լ. Գոգյան
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	Մ. Գ. Գրիգորյան
	ֆիզ.-մաթ. գիտ. թեկնածու
	Լ. Ն. Գալոյան
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The topic of the thesis was approved in Yerevan State University

Scientific advisor	Doctor of phys.-math. sciences
	S. L. Gogyan
Official opponents	Doctor of phys.-math. sciences
	M. G. Grigoryan
	Candidate of phys.-math. sciences
	L. N. Galoyan
Leading organization	Tbilisi State University

The defense will be held on July 23, 2019 at 15 : 00 at a meeting of the specialized council of mathematics 050, operating at the Yerevan State University (0025, 1 Alek Manukyan St, Yerevan).
The thesis can be found at the YSU library.
The synopsis was sent on June 14, 2019.

Scientific secretary
of specialized council T. N. Harutyunyan

Overview

Relevance of the topic. In a Banach space with Schauder basis it is natural to approximate a function by the terms of its series expansion. For example, we can approximate a function by partial sums of its series. In early studies, some linear approximations were obtained. But in recent years, with the rise of interest in problems related to Big Data and high resolution images, the usefulness and effectiveness of non-linear approximations became apparent. In late 1990s, greedy algorithms found popularity. These algorithms are used in applications in image/signal/data processing and in designing neural networks. Important results have been obtained by R. DeVore, S. Konyagin, V. Temlyakov, P. Wojtaszczyk, M. Grigoryan, among others. In this dissertation, we study some interesting problems related to Thresholding Greedy Algorithm.

Goals.

1. Improve the estimate for the quasi-greedy constant for quasi-greedy subsystems of the Haar system in $L^1(0,1)$.
2. Improve the estimate for the democratic constant for domcatic subsystems of the Haar multidimensional system in $L_1(0,1)^d$.
3. Study the behavior of the greedy algorithm, after modifing function on a small set.

Research methods. Methods of theory of funcions and real analysis.

Scientific novelty. All results are new.

Theoretical and practical value. All the results and developed methods represent theoretical interest.

Approbation of results. Most of the results were reported in the following conferences:

in seminars of the chair of Mathematical Analysis, "Annual sessions of the Armenian Mathematical Union (AMU)" - 2016 (Yerevan, Armenia). "Harmonic analysis and approximations, VII" - 2018 (Tsaghkadzor, Armenia), "Mergelyan-90" -2018 (Yerevan, Armenia).

Publications. The main results of this thesis are published in 6 works (3 papers and 3 conference abstracts), which are listed at the end of references.

Structure of the thesis. The thesis consists of introduction, two chapters, conclusion and bibliography with 35 items.

The content of the work

For any basis $\Psi = \{\psi_k\}_{k=1}^{+\infty}$ in the Banach space X and for any $f \in X$, we have the following expansion:

$$f = \sum_{k=1}^{+\infty} c_k(f, \Psi)\psi_k. \tag{1}$$

Assuming that $\{\|\psi_k\|_X\}_{k=1}^{+\infty}$ is bounded we may conclude that where $\lim c_k(f, \Psi) = 0$. Let $\Lambda_0 = \emptyset$, and by induction, for $m = 1, 2, 3, \ldots$ define the sequence of sets of positive integers Λ_m such that $| \Lambda_m |= m$, $\Lambda_{m-1} \subset \Lambda_m$, and

$$\min_{k\in\Lambda_m} | c_k(f, \Psi) |\geq \max_{k\notin\Lambda_m} | c_k(f, \Psi) | . \tag{2}$$

Note that the sets Λ_m may be not uniquely determined, but for the purposes of our thesis, it is not essential. We remark that the theorems presented below hold for any choice of Λ_m satisfying condition (2). We call the function

$$G_m(f) = \sum_{k\in\Lambda_m} c_k(f, \Psi)\psi_k,$$

the m-term greedy approximant of the function f with respect to the system Ψ, see [29] and [22] for more details. The approximation method which uses greedy approximants is called a Thresholding Greedy Algorithm (TGA).

There are several generalizations for Thresholding Greedy Algorithm. Here we give the definition of Weak Thresholding Greedy Algorithm with weakness parameter t. We put $\Lambda_0^t = \emptyset$ and by induction, for $m = 1, 2, 3, \ldots$ define the sequence of sets of positive integers Λ_m^t such that $| \Lambda_m^t |= m$, $\Lambda_{m-1}^t \subset \Lambda_m^t$, and

$$\min_{k\in\Lambda_m^t} | c_k(f, \Psi) |\geq t \max_{k\notin\Lambda_m^t} | c_k(f, \Psi) | . \tag{3}$$

Note, that when we put $t = 1$ we get definition of Λ_m. Denote

$$G_m^t(f) = \sum_{k\in\Lambda_m^t} c_k(f, \Psi)\psi_k,$$

To understand the effectiveness of the approximaion with a greedy algorithm we compare its accurancy with the best m-term approximation error using the basis vectors. We denote the best m-term approximation as follows,

$$\sigma_m(f, \Psi) = \inf_{\substack{\alpha_1, \alpha_2, \ldots, \alpha_m \\ k_1, k_2, \ldots k_m}} \|f - \sum_{i=1}^{m} \alpha_i \Psi_{k_i}\|. \tag{4}$$

Definition. (S. Dilworth, N. Kalton, D. Kutzarova, V. Temlyakov) The basis Ψ is called a greedy basis in X, if there exists $C \geq 1$ such that for any $m \in \mathbb{N}$ and $f \in X$ the following relation holds:

$$\|f - G_m(f, \Psi)\|_X \leq C\sigma_m(f). \tag{5}$$

It is also natural to compare the error of the TGA with the best m-term approximation using function expansion terms. We denote by $\tilde{\sigma}_m(f, \Psi)$ the following quantity:

$$\tilde{\sigma}_m(f, \Psi) = \inf_{k_1, k_2, \ldots k_m} \|f - \sum_{i=1}^{m} c_{k_i}(f) \Psi_{k_i}\|. \tag{6}$$

Definition. (S. Dilworth, N. Kalton, D. Kutzarova, V. Temlyakov) The basis Ψ is called an almost greedy basis in X, if there exists $C \geq 1$ such that for any $m \in \mathbb{N}$ and $f \in X$ the following relation holds:

$$\|f - G_m(f, \Psi)\|_X \leq C\tilde{\sigma}_m(f). \tag{7}$$

The authors of [22] show that any greedy basis can be characterized as an unconditional basis with the additional property of being democratic. Here, we give a slightly more general definition of democratic systems.

Definition. (V. Temlyakov, S. Konyagin) Let $\Psi = \{\psi_k\}_{k=1}^{+\infty}$ be a normalized system in X. Then Ψ is called democratic in X if and only if there exists a constant C such that for any two finite subsets A, B of positive integers with equal number of elements, i.e. $\mid A \mid = \mid B \mid$, the following relation holds:

$$\|\sum_{i \in A} \psi_i\|_X \leq C \|\sum_{i \in B} \psi_i\|_X. \tag{8}$$

The smallest C for which this inequality holds is called the democratic constant for Ψ.

Theorem A (V. Temlyakov, S. Konyagin, [22]) *The basis is a greedy basis in X if and only if it is unconditional and democratic in X.*

In order to characterize almost greedy bases, we introduce the following definition of quasi-greedy bases.

Definition. (V. Temlyakov, S. Konyangin) A basis $\Psi = \{\psi_k\}_{k=1}^{+\infty}$ is called a quasi-greedy basis of X if there exists a constant C such that for any $f \in X$ and for any $m \in \mathbb{N}$, we have

$$\|G_m(f)\| \leq C \cdot \|f\|. \tag{9}$$

The smallest value of C for which the inequality holds is called the quasi-greedy constant for the system Ψ.

Theorem B (P. Wojtaszczyk, [31]) *The basis Ψ in the Banach space X is a quasi-greedy basis if and only if for any element $f \in X$ we have*

$$\lim_{m\to\infty} \|f - G_m(f, \Psi, \Lambda)\|_X = 0. \tag{10}$$

If Ψ is an unconditional basis then for every permutation of natural numbers π we have that the $\sum_{k=1}^{+\infty} c_{\pi(k)}(f, \Psi)\psi_{\pi(k)}$ converges, so we can conclude that the unconditional bases are quasi-greedy. On the other hand, any quasi-greedy basis is not necessarily an unconditional basis. Morever, in [29], Temlyakov constructed a basis which is quasi-greedy and democratic, while it is not an unconditional basis.

The following theorem characterizes almost greedy bases.

Theorem C (S. Konyagin, V. Temlyakov, [22]) *The basis is an almost greedy basis in X if and only if it is quasi-greedy and democratic in X.*

It is shown in [23] that the weakness parameter t can't break the convergence for quasi-greedy bases, namely the following 2 theorems are proved there.

Theorem D(S. Konyagin, V. Temlyakov, [23]) itLet Ψ is a quasi-greedy basis in the Banach space X. Then for any $0 < t \leq 1$ and $f \in X$ the following holds:

$$\lim_{m\to\infty} \|f - G_m^t(f, \Psi)\|_X = 0.$$

Theorem E(S. Konyagin, V. Temlyakov, [23]) itLet Ψ is a quasi-greedy basis in the Banach space X. Then for any $0 < t \leq 1$ there exists a constant $C(t) \geq 1$ such that, for any $f \in X$ the following holds:

$$\|G_m^t(f, \Psi)\|_X \leq C(t)\|f\|_X$$

for any positive m.

It is well known from [4] that the Haar system is not quasi-greedy, so it is natural to investigate the subsystems of the Haar system. The authors of [4] also give an example of a quasi-greedy Haar subsystem. Later, Gogyan [9] characterizes all the quasi-greedy subsystems of the Haar system in $L_1[0,1]$. To formulate this result we first recall the definition of the Haar system.
We denote $\Delta_1 = \Delta_0^{(0)} = [0,1]$. For $j = 1, 2, \ldots, 2^i$ and $i = 0, 1, 2, \ldots,$ the intervals

$$\Delta_{2^i+j} = \Delta_i^{(j)} = \left[\frac{j-1}{2^i}, \frac{j}{2^i}\right),$$

are called dyadic intervals. Let us donate the set of all dyadic intervals by $\mathcal{D}$. Every dyadic interval is a union of two dyadic intervals, namely $\Delta_n = \Delta_{2n-1} \cup \Delta_{2n}$. These two intervals are called the left and right halves of the interval Δ_n, respectively. To each dyadic interval corresponds exactly one function of the Haar system, which is given by

$$h_n(t) = h_i^{(j)}(t) = h_{\Delta_n(t)} = \begin{cases} 2^i, & t \in \Delta_{2n-1}, \\ -2^i, & t \in \Delta_{2n}, \\ 0, & \text{otherwise.} \end{cases}$$

We also assume $h_1 = h_{\Delta_1} \equiv 1$. The set of the functions $H = \{h_n\}_{n=1}^{\infty}$ is called a Haar system.

For the set $\mathcal{A}$, $\mathcal{A} \subset \mathcal{D}$ we denote by $H_{\mathcal{A}} = \{h_{\mathcal{I}}\}_{\mathcal{I} \in \mathcal{A}}$ and $L_{\mathcal{A}} = \overline{span(H_{\mathcal{A}})}$, where the closure is taken with the $L_1[0,1]$-norm. For any $\mathcal{I}, \mathcal{J} \in \mathcal{D}$ with $\mathcal{J} \subset \mathcal{I}$, denote

$$C(\mathcal{I}, \mathcal{J}) = \{\Delta \in \mathcal{D} : \mathcal{J} \subseteq \Delta \subseteq \mathcal{I}\}. \tag{11}$$

$C(\mathcal{I}, \mathcal{J})$ is called a chain [8]. The length of the chain $C(\mathcal{I}, \mathcal{J})$ is the number of elements in the chain. Denote $H(\mathcal{A})$ the length of the longest chain in subsystem $\mathcal{A}$, see [8]. The following theorem was proved in [8].

Theorem F (S. Gogyan, [8]) For the set $\mathcal{A}$, $\mathcal{A} \subset \mathcal{D}$, the Haar subsystem $H_{\mathcal{A}}$ is a quasi-greedy basis in $L_{\mathcal{A}}$ if and only if $H(\mathcal{A}) < +\infty$.

Furthermore, an estimate $G(\mathcal{A}) \leq 2^{H+2}$ is obtained for a quasi-greedy constant in [8]. In Chapter 1, we improve the estimate for the quasi-greedy constant $H_{\mathcal{A}}$.

Theorem 1. *Assume there exists a subsystem of a Haar system $H_{\mathcal{A}}$ which is a quasi-greedy subsystem in $L_1[0,1]$. Then, for the quasi-greedy constant $G_{\mathcal{A}}$, we have the following estimate,*

$$\frac{H(\mathcal{A})}{16} \leq G_{\mathcal{A}} \leq 2H(\mathcal{A}) + 1.$$

The democratic subsystems of the 1-dimensional and multidimensional Haar systems in $L_1[0,1]$ and $L_1[0,1]^d$ are characterized respectively in [6] and [5]. We consider the multidimensional Haar system whose elements have cubic supports, and we first recall the definition of this system. The dyadic interval is the interval of type $\left[\frac{j-1}{2^n}, \frac{j}{2^n}\right)$, with $1 \leq j \leq 2^n$, $n \geq 0$. For a dyadic interval $\mathcal{I} = \left[\frac{j-1}{2^n}, \frac{j}{2^n}\right) \subset [0,1)$, we write

$$r_{\mathcal{I}}^{(0)}(t) = \begin{cases} \frac{1}{|\mathcal{I}|} & : \ t \in \mathcal{I} \\ 0 & : \ t \notin \mathcal{I} \end{cases}, \qquad r_{\mathcal{I}}^{(1)}(t) = \begin{cases} \frac{1}{|\mathcal{I}|} & : \ t \in \left[\frac{j-1}{2^n}, \frac{2j-1}{2^{n+1}}\right) \\ -\frac{1}{|\mathcal{I}|} & : \ t \in \left[\frac{2j-1}{2^{n+1}}, \frac{j}{2^n}\right) \\ 0 & : \ t \notin \mathcal{I} \end{cases},$$

For dyadic intervals $\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_d$ of the same length, the cube

$$\mathcal{I} = \mathcal{I}_1 \times \mathcal{I}_2 \times \ldots \mathcal{I}_d \tag{12}$$

is called a dyadic cube. By $\mathcal{D}^d$ we denote the set of all dyadic cubes of dimension d. To remind definition of multidimensional Haar system we need one more notation. Denote

$$\mathbb{M} = \mathcal{D}^d \times \{1, 2, \ldots, 2^d - 1\}. \tag{13}$$

To each element $(\mathcal{I}, j) \in \mathbb{M}$ corresponds one element of the multidimensional Haar function $h_{\mathcal{I}}^{(j)}$, which is defined in the following way

$$h_{\mathcal{I}}^{(j)}(x) = \prod_{i=1}^{d} r_{\mathcal{I}_k}^{(\epsilon_k)}(x_k), \tag{14}$$

where $x = (x_1, x_2, \ldots, x_d) \in [0,1(]^d$ and numbers $\epsilon_k \in \{0,1\}$ are defined by the representation $j = \sum_{k=1}^{d} \epsilon_k 2^{d-k}$. The set of functions $h_{\mathcal{I}}^{(j)}$ together with $h_{[0,1)^d}^{(0)} \equiv 1$ is a multidimensional Haar system.

Now, for any $\mathcal{I}, \mathcal{J} \in \mathcal{D}^d$ with $\mathcal{J} \subset \mathcal{I}$, denote

$$C(\mathcal{I}, \mathcal{J}) = \{\Delta \in \mathcal{D}^d : \mathcal{J} \subseteq \Delta \subseteq \mathcal{I}\} \tag{15}$$

which is called a chain [8]. The length of the chain $C(\mathcal{I}, \mathcal{J})$ is the number of elements in the chain. Also we will say that $\mathcal{J}$ is a son of $\mathcal{I}$ iff $\mathcal{J} \subset \mathcal{I}$ and $\mu(\mathcal{J}) = 2^{-d}\mu(\mathcal{I})$. By the term *complete chain* $C^d(\mathcal{I}, \mathcal{J})$ we mean the following set

$$C^d(\mathcal{I}, \mathcal{J}) = \{(\Delta, k) \in \mathbb{M} \ : \ \mathcal{J} \subseteq \Delta \subseteq \mathcal{I}, \ 1 \leq k \leq 2^d - 1\}. \tag{16}$$

By length of chain $C^d(\mathcal{I}, \mathcal{J})$ we mean the length of the chain $C(\mathcal{I}, \mathcal{J})$. Also, for $\mathcal{S} \subset \mathbb{M}$ denote by $G_{\mathcal{S}}$ the quasi-greedy constant constant for the system $\{h_{\mathcal{I}}^{(j)}\}_{(\mathcal{I},j)\in\mathcal{S}}$ in $L_1[0,1]^d$. We also put $H(\mathcal{S})$ the length of the longest chain in subsystem $\mathcal{S}$.

The proof of [5, Theorem 1] implies that the democratic constant satisfies the condition $D(\mathcal{S}) \leq 2^{(H-1)d}$, where $\mathcal{S}$ is the set of the dyadic cubes and H is the length of the longest full chain of the subsystem. In CHAPTER 1, we improve this result by proving the following theorem.

Theorem 2. *Let $\mathcal{S} \subset \mathbb{M}$ and let $\mathcal{S}$ contains the complete chains having maximal length H. Then $D(\mathcal{S}) < 2^d(2^d - 1)(H + 1)$.*

In fact the Thresholding Greedy Algorithm is rearranging the terms of the expansion in a decreasing order (by absolute value). It has been shown in [4] that the Haar system is not a quasi-greedy basis in $L_1[0,1]$. To overcome such kind of problems, one can consider modifying a function in a small set. The first steps in terms of modifications of functions is done by N. Luzin. He proved the following fundamental theorem.

Theorem G (N. Luzin, [24]) *For any measurable, almost everywhere finite in $[0,1]$ function f and any $\epsilon > 0$ there exists a measurable set E with $|E| > 1 - \epsilon$ and a continuous function g in $[0,1]$ which coincids with f on E.*

Later, some interesting results on correction were obtained by Menshov E. [25], [26], A. Talalyan [28], W. Price [27], M. Grigoryan [16], [13], [14], Arutiunyan F. [21] and others.

The following two theorems indicate that in the case of the Haar system, we can modify the function in a small set such that its non-zero terms will be in a decreasing order, while we are not able to order all the terms monotonically.

Theorem H (S. Gogyan, M. Grigoryan, [18]) *For any* $0 < \epsilon < 1$, *there exists a measurable set* $E \subset [0,1]$ *with* $|E| > 1 - \epsilon$, *such that for any function* $f \in L_1[0,1]$ *there is some* $\tilde{f} \in L_1[0,1]$ *coinciding with* f *on* E *and all non-zero terms of the sequence* $\{c_n(\tilde{f})\}$ *are arranged in a decreasing order.*

Theorem I (S. Gogyan, M. Grigoryan , [18]) *For any mesurable set* $E, E \subset [0,1]$ *with* $0 < |E| < 1$ *there exists a function* $f_0 \in L_1[0,1]$ *such that if* $f \in L_1[0,1]$ *coinciding with* f_0 *on* E, *then the sequence* $\{|c_n(f)|\}_{n=1}^{\infty}$ *cannot be monotonically decreasing, where* $c_n(f)$ *is the* n*-th coefficient of the Haar system normalized in* $L_1[0,1]$.

The analogous questions were considered for other bases and in other spaces. For the space $C(0,1)$ of continuous on $[0,1]$ functions and for Faber-Schauder system some interesting results are obtained by M. Grigoryan and A. Sargsyan in [20]. Let's remind the definition of Faber-Schauder system. Definition and notations are due [20].

The Faber-Schauder system is the sequence of functions $\Phi = \{\phi_n\}_{n=0}^{+\infty}$, defined on segment $[0,1]$, in which $\phi_0 \equiv 1$, $\phi_1(x) = x$ on $[0,1]$ and for $n = 2^k + i$, $k = 0,1,2,\ldots$, $i = 1,2,\ldots,2^k$ one has

$$\phi_n(x) = \phi_k^{(i)}(x) = \begin{cases} 0, & x \notin [\frac{i-1}{2^k}, \frac{i}{2^k}], \\ 1, & x = \frac{2i-1}{2^{k+1}}, \\ \text{is} & \text{linear and continious on} [\frac{i-1}{2^k}, \frac{2i-1}{2^{k+1}}] \text{ and } [\frac{2i-1}{2^{k+1}}, \frac{i}{2^k}] \end{cases}$$

The following definition is also due [20].

Definition *Let* Ψ *is a basis in a Banach space* X *and let* $0 < t \leq 1$. *We say that coefficients of* $f \in X$ *are* t*-monotone with respect to* Ψ *iff for any integer* $1 \leq n < m$ *one has either*

$$c_m(f) = 0,$$

or

$$| c_n(f) | \geq t \cdot | c_m(f) | .$$

The following theorem is proved in [20].

Theorem J (M. Grigoryan, A. Sargsyan) *For every* $\epsilon \in (0,1)$ *there exists a measurable set* $E \subset [0,1]$ *with measure* $|E| < 1-\epsilon$*, such that to each function* $f \in C[0,1]$ *one can find a function* $g \in C[0,1]$ *that coincides with* f *on* E *and coefficients of which's expansion by the Faber-Schauder system are t-monotone for all* $t \in (0, \frac{1}{2})$]. Also, in 2013 the following theorem was proved.

Theorem K (M. Grigoryan, V. Krotov, [19]) *Let* $\{a_n\}$ *is a decreasing sequence of real numbers such, that*

$$\lim_{n\to\infty} a_n = 0,$$

and

$$\sum_{n=1}^{\infty} \frac{a_n}{n} = +\infty.$$

Then for any ϵ *with* $0 < \epsilon < 1$ *and for any measurable and almost everywhere finite (on* $[0,1]$*) function* f *there exists a function* $\tilde{f} \in C[0,1]$ *with the following properties:*

i) $\mu\{\tilde{f} \neq f\} < \epsilon$,

ii) $a_n = c_n(\tilde{f})$ *for all* n *with* $c_n(\tilde{f}) \neq 0$*, where* $c_n(\tilde{f})$ *is the n-th Faber-Schauder series coefficient of function* $\tilde{f}$.

In Chapter 2, we prove the following result.

Theorem 3. *There exists a normalized basis* $\Psi = \{\psi_n\}_{n=1}^{\infty}$ *such that for any* $\epsilon, 0 < \epsilon < 1$ *there exists a measurable set* $E \subset [0,1]$ *with* $|E| > 1-\epsilon$ *such that for every function* $f \in L_1[0,1]$ *we can find a function* $\tilde{f}, \tilde{f} \in L_1[0,1]$ *which coincides with* f *on* E *and all the terms of* $\{c_n(\tilde{f})\}$ *are arranged in a decreasing order.*

At the end we show the basis, which has that property.

Let us split the Haar system into two subsequences, $\{b_i\}$ and $\{\phi_i\}$, where $b_i = h_i^{(2)}, i = 1, 2, \dots$.

Next, denote $N_0 = 0$ and $N_i = \sum_{k=1}^{i} M_i = \sum_{k=1}^{i} 2^{2^k}$. [18] investigates a system of functions $F = \left\{\{f_{(i,j)}\}_{j=0}^{M_i}\right\}_{i=1}^{\infty}$ which is defined in the following way:

$$f_{(i,0)} = \phi_i - \frac{1}{M_i+1} \sum_{N_{i-1}+1 \le k \le N_i} b_k,$$

$$f_{(i,j)} = f_{(i,0)} + b_{N_{i-1}+j}, j = 1, 2, \dots, M_i.$$

It is noted in [11] that the system $\{f_{(i,j)}\}$ is a basis in $L_1[0,1]$. Here we remark some properties of the system F and numbers M_i, that are essential for our purposes.

$$2 \le \|f_{(i,j)}\| \le 3, \tag{17}$$

$$\phi_i = \frac{1}{M_i+1}\sum_{j=0}^{M_i} f_{(i,j)}, \tag{18}$$

$$\sum_{j=1}^{i} \frac{M_j}{M_{i+1}} < \frac{2}{M_i} < 2^{-i}. \tag{19}$$

In the proof of the Theorem 3 the system $F = \left\{\{f_{(i,j)}\}_{j=0}^{M_i}\right\}_{i=1}^{\infty}$ is used as an example of the basis having the required property.

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Articles

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2. Gogyan S. and Srapionyan N. *Modifications of Values of Functions on a Set of Small Measure and Series with Monotone Coefficients*, Dokl. Nats. Akad. Nauk Armen., **119**(2019), N1, pp. 29-32.

3. Srapionyan N. *On the Democratic Constant of Haar Subsystems in* $L_1(0,1)^d$, Armen. J. Math., **11**(2019), N8, pp. 1-6.

Conference Theses

4. Srapionyan N., Gogyan S., *On the quasi-greedy constant of the Haar subsytems in* $L_1(0,1)$. Annual session 2016 of the Armenian Mathematical Union (AMU), dedicated to 110th anniversary of Artashes Shahinyan, Yerevan, Armenia, May 30- June 1, p. 115, 2016.

5. Srapionyan N., *On the quasi-greedy constant of the Haar subsytems in* $L_1(0,1)$. Conference Dedicated to the Memory of Sergey Mergelyan, Tsaghkadzor, Armenia, May 20-25, p. 79, 2018.

6. Srapionyan N., Gogyan S., *On the quasi-greedy constant of the Haar subsytems in* $L_1(0,1)^d$. Harmonic Analysis and Approximations VII, dedicated to 90th anniversary of Alexandr Talalyan, Tsaghkadzor, Armenia, September 16- 22, p. 41, 2018.

Ամփոփում

Ատենախոսության մեջ ստացվել են հետևյալ հիմնական արդյունքները.

- Գնահատվել է Հաարի համակարգի քվազի-ագահ ենթահամակարգերի քվազի-ագահ ության գործակիցը $L^1(0,1)$-ում: Նախկին հայտնի էքսպոնենցիալ գնահատականը ճշգրտվել է գծայինով:

- Գնահատվել է նաև բազմաչափ Հաարի համակարգի դեմոկրատիկ ենթահամակարգերի դեմոկրատիկության գործակիցը $L_1(0,1)^d$-ում: Նախկին հայտնի էքսպոնենն- ցիալ գնահատականը ճշգրտվել է գծայինով:

- Գոյություն ունի բազիս, որ ցանկացած $f \in L^1(0,1)$ ֆունկցիա փոքր չափի վրա փոխելուց հետո նրա վերլուծության գործակիցները ըստ այդ բազիսի կլինի մոնոտոն նվազող հաջորդականություն:

Заключение

В диссертационной работе получены следующие результаты:

- Был оценен коэффициент квази-жадности в квази-жадной подсистеме системы Хаара в $L_1(0,1)$. Известный прежде экспоненциальный оценка был скорректирован по линейности.

- Был оценен также рейтинг демократичности в демократичной подсистеме многомерной системы Хаара в $L_1(0,1)^d$. Известный прежде экспоненциальный оценка был скорректирован по линейности.

- Существует базис, что коэффициенты разложения любой функции $f \in L_1(0,1)$ после изменений на малой меры будут монотонно убывающей последовательностью.