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ֆիզիկամաթեմատիկական գիտությունների թեկնածուի
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VACUUM QUANTUM EFFECTS IN CURVED SPACES INDUCED BY TOPOLOGY

Thesis for the degree of Candidate of physical and mathematical sciences
Speciality 01.04.02 - "Theoretical Physics"

ABSTRACT

YEREVAN - 2019

Ատենախոսության թեման հաստատվել է Երևանի պետական համալսարանում

Գիտական ղեկավար՝	ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Ա.Ա. Սահարյան
Պաշտոնական ընդդիմախոսներ՝	ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Գ.Ս. Հաջյան ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Լ.Շ. Գրիգորյան
Առաջատար կազմակերպություն՝	ՀՀ ԳԱԱ Վ. Համբարձումյանի անվան Բյուրականի աստղադիտարան

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GENERAL DESCRIPTION OF THE WORK

Relevance of topic. The prediction of nontrivial properties of the vacuum state is among the most interesting results in quantum field theory. The vacuum is defined as a state of a quantized field with zero number of quanta. By taking into account that the field operator does not commute with the operator of the number of quanta, from here it follows that in the vacuum state the field is subject to quantum fluctuations. The properties of the vacuum state are manifested in its response under external influences, such as external electromagnetic and gravitational fields. In addition, for a large number of physical problems one needs to consider a quantum field in backgrounds having boundaries. The boundaries may have various physical nature: surfaces of conductors and dielectrics in quantum electrodynamics, the surfaces separating two different phases, domain walls, different types of horizons in gravitational physics, branes in string theory and in braneworld scenarios. The presence of boundaries gives rise to the modification of the spectrum for vacuum fluctuations of quantum fields. As a consequence of that the vacuum expectation values of physical observables are shifted by an amount depending on the geometry of boundaries and on boundary conditions imposed on the field operator. This effect is referred as the Casimir effect. It has been investigated for a large number of boundary geometries [1,2] and has been confirmed by high-precision experiments.

A similar effect, referred as topological Casimir effect, arises when the boundary conditions originate from the nontrivial spatial topology. A number of fundamental theories are formulated in backgrounds with nontrivial topology (Kaluza-Klein theories, supergravity, string theories) and it is of interest to investigate the properties of the vacuum state for the corresponding geometries. For example, the string theories are formulated in background of $(1+9)$-dimensional spacetimes and it is assumed that the six spatial dimensions are compactified. As a consequence of that the background geometry has nontrivial topology. The geometries with compact spatial dimensions may play also an important role in inflationary models of the early expansion of the Universe [3], by providing proper initial conditions for the start of accelerated expansion. The physical effects arising as a consequence of compactification of spatial dimensions include topological mass generation, instabilities in interacting field theories and symmetry breaking. The topological Casimir effect provides a mechanism for stabilization of moduli fields in models with extra dimensions. Those fields are related to the lengths of compact dimensions and their expectation values determine the values of effective physical constants. In order to escape the variations of the constants, the expectation values should be stabilized. The Casimir energy related to the nontrivial spatial topology can also play the role of dark energy driving the accelerated expansion of the universe at recent epoch. The topological issues also play an important role in effective theories describing a number of condensed matter systems [4,5].

Among the most important directions in the investigation of both the boundary-induced and topological Casimir effects is the dependence of the properties of the vacuum on the local geometry of background spacetime. These investigations require the knowledge of complete set of modes for quantum fields in those geometries, obeying the boundary and periodicity conditions. These modes can be explicitly found for highly symmetric background spacetimes only. In particular, the investigation of quantum effects in maximally symmetric spacetimes has attracted a great deal of attention. This allows to form an idea of the effect of the gravitational field on various physical processes for more general background geometries. The maximally symmetric spacetimes are solutions of the Einstein equations with zero (Minkowski spacetime), positive (de Sitter spacetime) and negative (anti-de Sitter spacetime) cosmological constants.

Because of the high symmetry, a large number of problems in quantum field theory are exactly solvable on those backgrounds. The corresponding effects in de Sitter (dS) and anti-de Sitter (AdS) spacetimes include the vacuum polarization and the creation of particles in gravitational fields. These effects play an important role in the early Universe cosmology, in black hole physics and in braneworld models with extra dimensions.

The aim of the thesis is to investigate the influence of the gravitational field and nontrivial spatial topology on the vacuum expectation values of physical observables for scalar, fermionic and electromagnetic fields. We have considered:

- Mean field squared and the vacuum expectation value of the energy-momentum tensor for a charged scalar field in dS spacetime with a part of spatial dimensions compactified to a torus.
- Casimir densities for a charged scalar field induced by a plate with Robin boundary condition in dS spacetime with compact dimensions.
- Vacuum expectation value of the current density for a charged fermionic field in locally AdS spacetime with a compact subspace in the presence of a brane and abelian gauge field.
- Boundary-induced quantum effects for the electromagnetic field in AdS spacetime for a general case of spatial dimension.

Scientific novelty. Closed analytic expressions are derived for the vacuum expectation values of the scalar field squared and of the energy-momentum tensor in dS spacetime having compact dimensions for general values of the phases in periodicity conditions. The expectation values are periodic functions of the magnetic flux enclosed by compact dimensions. For lengths of compact dimensions smaller than the curvature radius of the background geometry, the behavior of the vacuum characteristics is essentially different from that for the locally Minkowski bulk. The influence of planar boundary with Robin boundary condition is investigated on the vacuum expectation values. It is shown that the effects of gravity are essential at distances from the boundary larger than the dS curvature radius. As a consequence of the time-dependence of the background metric, the vacuum energy-momentum tensor has a nonzero off-diagonal component that describes energy flux along the direction perpendicular to the boundary. The complete set of modes are obtained for a Dirac spinor field in AdS spacetime with compact spatial dimensions in the presence of a brane parallel to the AdS boundary. By using these modes, the vacuum expectation value of the fermionic current density is investigated. The corresponding results are applied to higher-dimensional generalizations of the Randall-Sundrum type braneworld models with compact dimensions and to electronic subsystem in curved graphene tubes, described by the Dirac model in the long-wavelength approximation. The electromagnetic field vacuum fluctuations are investigated in AdS spacetime in the presence of a boundary parallel to the AdS horizon. The boundary-induced contributions in the vacuum energy density and stresses are exponentially suppressed at distances from the boundary larger than the AdS curvature radius. This is in clear contrast to the case of the Minkowski bulk where the decay of the Casimir densities, as functions of the distance, is as power-law.

Practical importance. The Hadamard function for a charged scalar field in the geometry of a plate in dS spacetime can be applied for the investigation of other characteristics of the vacuum state, such as the expectation values of the charge and current densities. The mode functions for a fermionic field can be used for the investigation of the finite temperature effects on the current density in AdS spacetime with compact dimensions and also for the evaluation of the expectation value of the energy-momentum tensor. The results for the fermionic current density can be applied to other planar condensed matter systems, described in the long-wavelength approximation by the Dirac model.

Basic results to be defended:

1. Closed expressions are derived for the topological contributions in the vacuum expectation values of the field squared and energy-momentum tensor for a charged scalar field in dS spacetime with a part of spatial dimensions compactified on a torus. Depending on values of the phases in the quasiperiodicity conditions along compact dimensions, the vacuum energy density and stresses can be either negative or positive.

2. The boundary-induced parts in the mean field squared and in the vacuum expectation value of the energy-momentum tensor are explicitly extracted in the geometry of planar boundary in dS spacetime with compact dimensions. These parts dominate near the boundary and in that region the effects of gravity are weak. The influence of the gravitational field is essential at distances from the boundary larger than the dS curvature scale. The vacuum energy-momentum tensor has an off-diagonal component that corresponds to energy flux normal to the boundary.

3. The fermionic modes are found in locally AdS spacetime with compact dimensions in the presence of a brane parallel to the AdS boundary and of a constant gauge field. By using these modes, the effects of a brane on the fermionic current density are investigated. The direction of the current density depends on the magnetic flux enclosed by compact dimensions. Near the AdS horizon the total current is dominated by the part corresponding to the geometry in the absence of the brane. The current density vanishes on the AdS boundary and is finite on the brane. Applications are given to Z_2-symmetric braneworld models. Two types of boundary conditions for fermionic fields on the brane, imposed by the Z_2-symmetry, are specified. The results for a three-dimensional spacetime are applied for the investigation of the edge effects on the ground state current density induced in curved graphene tubes.

4. The effects of gravity and a planar boundary on the local properties of the electromagnetic vacuum are investigated in AdS spacetime with an arbitrary number of spatial dimensions. The boundary-induced contribution in the vacuum expectation value of the energy-momentum tensor is separated. In spatial dimensions larger than three, at large distances from the boundary these contributions, as functions of the proper distance, are exponentially small.

Approbation of the work. The results of the thesis were reported at the conferences "Modern Physics of Compact Stars and Relativistic Gravity" (Yerevan, 2015), "The XXIII International Scientific Conference of Young Scientists and Specialists 2019" (Dubna, Russia, 2019) and have been discussed at the seminars of the Chair of Theoretical Physics of Yerevan State University and of the INFN National Laboratory of Frascati (Frascati, Italy).

Publications. Six papers are published on the topic of the thesis.

Structure of the thesis. The thesis consists of Introduction, four Chapters, Conclusion and the list of references. It contains 125 pages, including 19 figures.

CONTENT OF THE THESIS

In **Introduction** the scientific literature related to the topic of the thesis is reviewed, the relevance of the topic is argued, the aim of the work, the scientific novelty and the practical value are presented, the basic results to be defended are described.

In **Chapter 1** the vacuum polarization for a charged scalar field is investigated in locally dS spacetime with toroidally compactified spatial dimensions. The background geometry is described by the line element

$$ds^2 = dt^2 - e^{2t/\alpha} \sum_{l=1}^{D} (dx^l)^2. \tag{1}$$

For the coordinates $\mathbf{x}_p = (x^1, ..., x^p)$, as usual, we take $-\infty < x^l < \infty$, $l = 1, \ldots, p$, whereas the remaining coordinates $\mathbf{x}_q = (x^{p+1}, ..., x^D)$, with $q = D - p$, are compactified to a torus $T^q = (S^1)^q$ with the lengths of the compact dimensions L_l: $0 \le x^l \le L_l$, $l = p+1, \ldots, D$. Hence, for the spatial topology one has $R^p \times T^q$. In terms of the conformal time $\tau = -\alpha e^{-t/\alpha}$, $-\infty < \tau \le 0$, the line element (1) is written in a conformally-flat form. The corresponding field equation reads

$$\left(D_\mu D^\mu + m^2 + \xi R\right) \varphi(x) = 0, \tag{2}$$

where $R = D(D+1)/\alpha^2$ is the Ricci scalar of the background spacetime. We assume the presence of a classical abelian gauge field A_μ and the gauge extended covariant derivative operator is given by $D_\mu = \nabla_\mu + ieA_\mu$, with e being the charge of the field quanta. The topology is nontrivial and it is required to specify the periodicity conditions for the field operator along compact dimensions. We consider generic quasiperiodic boundary conditions with general phases α_l, $l = p+1, ..., D$:

$$\varphi(t, \mathbf{x}_p, \mathbf{x}_q + L_l \mathbf{e}_l) = e^{i\alpha_l} \varphi(t, \mathbf{x}_p, \mathbf{x}_q), \tag{3}$$

where $\mathbf{e}_l$ is the unit vector along the dimension x^l. For the gauge field we will assume that $A_\mu = \text{const}$. Though the corresponding field tensor vanishes, the nontrivial topology of the background spacetime gives rise to the Aharonov-Bohm type effect on the expectation values of physical observables. The shift in the phases due to the gauge field may be presented as $eA_l L_l = -2\pi \Phi_l / \Phi_0$, where Φ_l is the flux enclosed by the circle corresponding to the lth compact dimension and $\Phi_0 = 2\pi/e$ is the flux quantum.

In Section 1.2 the vacuum expectation value of the field squared is evaluated assuming that the field is in the Bunch-Davies vacuum state. The mean field squared is presented in the form $\langle \varphi\varphi^\dagger \rangle_{p,q} = \langle \varphi\varphi^\dagger \rangle_{dS} + \langle \varphi\varphi^\dagger \rangle_c$, where $\langle \varphi\varphi^\dagger \rangle_{dS}$ is the expectation value in the uncompactified dS spacetime and $\langle \varphi\varphi^\dagger \rangle_c$ is induced by the comactification to a torus. The divergences in the coincidence limit are contained in the part $\langle \varphi\varphi^\dagger \rangle_{dS}$ and the renormalization is required for that part only. The topological part $\langle \varphi\varphi^\dagger \rangle_c$ is an even periodic function of the magnetic fluxes Φ_l with the period equal to the flux quantum. It depends on the phases α_l and on A_l in the form of the combination $\tilde{\alpha}_l = \alpha_l + eA_l L_l$, $l = p+1, ..., D$. In the model with a single compact dimension x^D ($q = 1$, $p = D - 1$) the expression for the topological part is simplified to

$$\langle \varphi\varphi^\dagger \rangle_c = \frac{4\alpha^{1-D}\eta^D}{(2\pi)^{D/2+1}} \int_0^\infty dz\, z g_\nu(\eta z) \sum_{n=1}^\infty \frac{\cos(n\tilde{\alpha}_D)}{(nL_D)^{D-2}} f_{D/2-1}(nL_D z), \tag{4}$$

where $\eta = |\tau|$, $\nu = \left[D^2/4 - D(D+1)\xi - m^2\alpha^2\right]^{1/2}$, $g_\nu(x) = K_\nu(x)[I_{-\nu}(x) + I_\nu(x)]$ and $f_\mu(x) = x^\mu K_\mu(x)$, with $I_\nu(x)$ and $K_\nu(x)$ being the modified Bessel functions. For a conformally coupled scalar field, $\xi = \xi_c = (D-1)/(4D)$, the problem on dS bulk is conformally related to the problem on the locally Minkowski bulk with spatial topology $R^p \times T^q$. The result could be obtained from the corresponding result in the Minkowski bulk (denoted here by $\Delta_r \langle \varphi^2 \rangle_{p,q}^{(M)}$) by using the standard relation $\Delta_r \langle \varphi^2 \rangle_{p,q} = (\eta/\alpha)^{D-1} \Delta_r \langle \varphi^2 \rangle_{p,q}^{(M)}$. The topological contribution in the vacuum expectation value depends on η and L_i, $i = p+1, ...D$, through the ratio L_i/η. By taking into account that, for a fixed η the quantity $L_{(p)i} = \alpha L_i/\eta$ is the proper length of the ith compact dimension, we see that the ratio L_i/η is the proper length of the compact dimension measured in units of the dS curvature scale.

In Section 1.3 the vacuum expectation value of the energy-momentum tensor is investigated. Similar to the case of the field squared, it is decomposed as $\langle T_i^k \rangle_{p,q} = \langle T_i^k \rangle_{dS} + \langle T_i^k \rangle_c$, where $\langle T_i^k \rangle_{dS}$ is the vacuum energy-momentum tensor in uncompactified dS spacetime and $\langle T_i^k \rangle_c$ is induced by the comactification to a torus. From the maximal symmetry of the Bunch-Davies vacuum state it follows that $\langle T_i^k \rangle_{dS} = \mathrm{const} \cdot \delta_i^k$. The off-diagonal elements of the topological part $\langle T_i^k \rangle_c$ vanish. In the problem with a single compact dimension x^D the diagonal components are presented as (no summation over i)

$$\langle T_i^i \rangle_c = \frac{4\alpha^{-1-D}\eta^D}{(2\pi)^{D/2+1}} \int_0^\infty dz\, z g_\nu(\eta z) \sum_{n=1}^{\infty} \frac{\cos(n\tilde{\alpha}_D)}{(nL_D)^{D-2}} P^{(i)}(nL_D z), \tag{5}$$

where the functions $P^{(i)}(x)$ are linear combinations of the functions $f_{D/2-1}(x)$ and $f_{D/2}(x)$ with coefficients depending on x. In the Minkowski bulk the stresses along uncompactified dimensions are equal to the energy density. This property is a consequence of the invariance with respect to the Lorentz boosts along uncompact dimensions. For the dS bulk, the topological contributions to the stresses along uncompact dimensions, in general, do not coincide with the energy density.

In the left panel of figure 1, for the fixed value $L/\eta = 1$, we have plotted the topological contributions $\langle T_i^i \rangle_c$ to the vacuum expectation values versus $\tilde{\alpha}_D = \tilde{\alpha}$ for the $D = 4$ model with a single compact dimension. The numbers near the curves correspond to the value of the index i. For $L/\eta = 1$ the graph for the stress $\langle T_i^i \rangle_c$ is close to the curve for the energy density (this is not the case for larger values of L/η) and we have not presented it. The full and dashed curves correspond to $m\alpha = 0.5$ and $m\alpha = 2$, respectively. In particular, from the graphs we see that the sign of the topological contribution to the vacuum energy density can be controlled by tuning the magnetic flux enclosed by the compact dimension. In the right panel of figure 1 the dependence of the topological contribution to the energy density on L/η (proper length of the compact dimension measured in units of α) is displayed for the same model. The numbers near the graphs correspond to the value of $m\alpha$. For the full and dashed curves one has $\tilde{\alpha} = 0.2\pi$ and $\tilde{\alpha} = 0.9\pi$, respectively. For $m\alpha = 2$ the parameter ν is purely imaginary and the oscillatory decay of the vacuum expectation value for large values of L/η seen in accordance with the asymptotic analysis given above. In the limit $L/\eta \to 0$ the vacuum expectation values behave like $1/(L/\eta)^{D+1}$.

If the proper length of the compact dimension is much smaller than the dS curvature scale, the effects induced by gravity are small and the vacuum densities are related to the corresponding ones for a massless field in Minkowski spacetime with compact dimensions by the standard conformal relation. For a fixed values of the lengths of compact dimensions, this limit corresponds to the early stages of the cosmological expansion ($t \to -\infty$), and the topological Casimir densities behave as $e^{-(D+1)t/\alpha}$. The effect of gravity on the vacuum expectation values is essential for proper lengths of the compact dimensions larger than the dS curvature radius. In

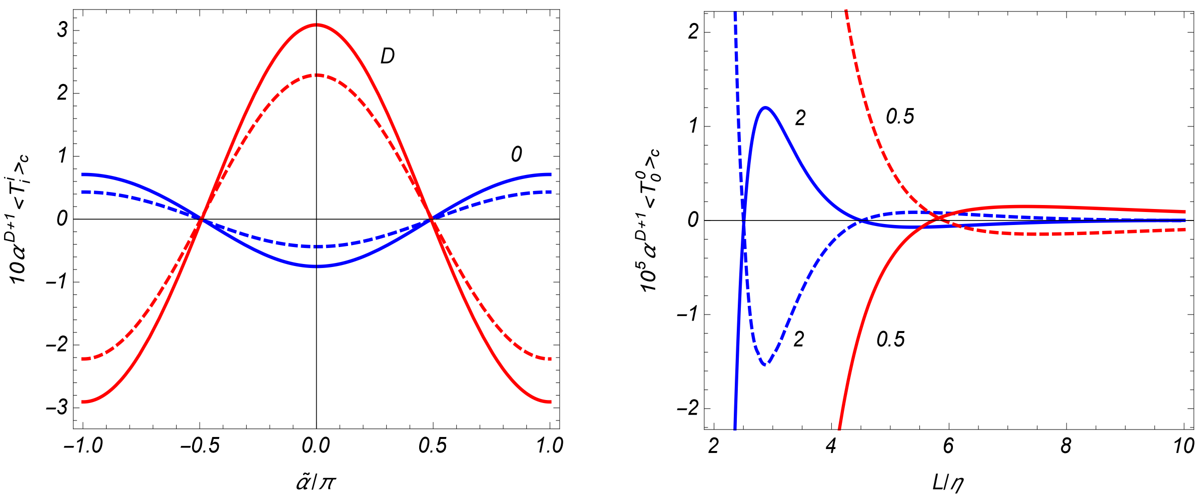

Figure 1: The topological parts in the vacuum energy-momentum tensor versus the phase $\tilde{\alpha}$ (left panel) and the topological parts in the vacuum energy density as a function of the ratio L/η (right panel) for the $D = 4$ model with a single compact dimension.

this limit and for positive values of the parameter ν, the topological parts ate suppressed by the factor $e^{-(D-2\nu)t/\alpha}$. Hence, in this limit the total vacuum expectation value is dominated by the uncompactified dS parts. For purely imaginary ν, the damping of the topological contributions is oscillatory with the amplitude decaying as $e^{-Dt/\alpha}$. For fixed values of the lengths L_l, the limit under consideration corresponds to the late stages of cosomological expansion, $t \to +\infty$.

In **Chapter 2** the influence of a planar boundary is investigated on the local properties of the vacuum state for a charged scalar field in locally dS spacetime with compact dimensions. Here the background geometry and the field equation are the same as that we have considered in the Chapter 1. The presence of a planar boundary at $x^p = 0$ is assumed on which the field obeys the Robin boundary condition

$$(1 + \beta D_p)\varphi = 0, \quad x^p = 0, \tag{6}$$

with a constant coefficient β. The properties of the vacuum state are encoded in two-point functions. As such the Hadamard function is taken. The latter can be split into two parts: $G^{(1)}(x, x') = G_0^{(1)}(x, x') + G_b^{(1)}(x, x')$, where $G_0^{(1)}(x, x')$ is the Hadamard function for locally dS spacetime with compact dimensions when the boundary at $x^p = 0$ is absent and $G_b^{(1)}(x, x')$ is induced by the boundary at $x^p = 0$. The boundary-induced contribution in the Hadamard function is presented in the form

$$\begin{aligned} G_b^{(1)}(x, x') &= \frac{(\eta\eta')^{D/2}}{2^p\pi^p V_q \alpha^{D-1}} \int d\mathbf{k}_{p-1} e^{i\mathbf{k}_{p-1}\Delta\mathbf{x}_{p-1}} \sum_{\mathbf{n}_q} e^{i\mathbf{k}_q\Delta\mathbf{x}_q} \int_k^\infty du e^{-u(x^p+x^{p\prime})} \frac{\beta u + 1}{\beta u - 1} \\ &\quad \times \{K_\nu(\eta y)[I_{-\nu}(\eta' y) + I_\nu(\eta' y)] + [I_{-\nu}(\eta y) + I_\nu(\eta y)]K_\nu(\eta' y)\}_{y=\sqrt{u^2-k^2}}, \end{aligned} \tag{7}$$

where $k = \sqrt{\mathbf{k}_{p-1}^2 + \mathbf{k}_q^2}$.

In Section 2.2 the vacuum expectation value of the field squared is investigated. Similar to the Hadamard function, the mean field squared is decomposed as $\langle\varphi\varphi^+\rangle = \langle\varphi\varphi^+\rangle_0 + \langle\varphi\varphi^+\rangle_b$, where $\langle\varphi\varphi^+\rangle_0$ is the vacuum expectation value in the dS spacetime with compact dimensions without boundaries, and $\langle\varphi\varphi^{+}\rangle_b$ is induced by the boundary at $x^p = 0$. The latter one is presented in the form

$$\langle\varphi\varphi^{+}\rangle_b = \frac{2^{1-p}\pi^{-(p+1)/2}}{\Gamma((p-1)/2)V_q\alpha^{D-1}}\sum_{\mathbf{n}_q}\int_0^{\infty} dx\, x^p f(\sqrt{x^2+k^2_{\mathbf{n}_q}}, x^p) h_\nu^{(0)}(\eta x), \tag{8}$$

where $h_\nu^{(0)}(\eta x) = \eta^D \int_0^1 dz\, z(1-z^2)^{(p-3)/2} g_\nu(\eta x z)$. For a conformally coupled massless scalar field the problem under consideration is conformally related to the corresponding problem in Minkowski spacetime with compact dimensions. The expression for the vacuum expectation value of the field squared is further simplified for special cases of Dirichlet and Neumann boundary conditions with the values $\beta = 0$ and $\beta = \infty$, respectively. The boundary-induced part in the vacuum expectation value of the field squared is a function of the ratio x^p/η. The latter is the proper distance from the boundary $\alpha x^p/\eta$ measured in units of the curvature radius α. It is shown that the influence of the gravitational field on boundary-induced contribution is weak for points close to the boundary, whereas at proper distances larger than the dS curvature radius the effect of gravity is crucial.

In Section 2.3, a decomposition for the expectation value of the energy-momentum tensor is provided, $\langle T_{ik}\rangle = \langle T_{ik}\rangle_0 + \langle T_{ik}\rangle_b$, and explicit expressions for the boundary-induced contributions are given. The expectation values $\langle T_i^k\rangle_b$ are even periodic functions of the magnetic fluxes enclosed by compact dimensions with the period equal to the flux quantum. They depend on L_l, β, x^p and η through the dimensionless ratios L_l/η, β/η, x^p/η, where L_l/η is the proper length of the compact dimension in units of the dS curvature scale α and x^p/η is the proper distance from the plate in the same units. Compared to the problem with the absence of the plate, an important difference is the appearance of the off-diagonal component

$$\langle T_0^p\rangle_b = \frac{2\eta A_p}{V_q\alpha^{D+1}}\sum_{\mathbf{n}_q}\int_0^{\infty} dx\, x^p\sqrt{x^2+k^2_{\mathbf{n}_q}} f(\sqrt{x^2+k^2_{\mathbf{n}_q}}, x^p)\left[\left(\xi-\frac{1}{4}\right)\eta\partial_\eta+\xi\right] h_\nu^{(0)}(\eta x), \tag{9}$$

It is induced by the plate and corresponds to an energy flux along the direction perpendicular to the plate. For a conformally coupled massless field the energy flux vanishes. This result could also be directly obtained on the base of that for a conformally coupled field the problem in locally dS spacetime is conformally related to the corresponding problem in locally Minkowskian spacetime and in the latter problem the energy flux is zero. Depending on the values of the parameters, the flux can be either positive or negative, corresponding to the flux direction from or to the boundary, respectively. In the early stages of the cosmological expansion, corresponding to large values of the parameter η, the flux density $\langle T_0^p\rangle$ behaves as $(\eta/\alpha)^D$. At late stages of the expansion the parameter η is small and the behavior of the flux density is essentially different for positive and imaginary values of ν. In the first case, corresponding to relatively small values of $m\alpha$ (the mass of the field quanta measured in units of the inverse curvature scale $1/\alpha$), the flux density decays monotonically, like $(\eta/\alpha)^{D+1-2\nu}$, $\eta \to 0$. For purely imaginary ν, the decay is oscillatory.

In **Chapter 3**, the properties of the vacuum state for a charged fermionic field $\psi(x)$ are investigated in locally AdS spacetime with an arbitrary number of toroidally compactified spatial dimensions. The line element is given by

$$ds^2 = e^{-2y/a}\eta_{ik}dx^i dx^k - dy^2 = (a/z)^2\left(\eta_{ik}dx^i dx^k - dz^2\right),\ z = ae^{y/a}, \tag{10}$$

where $i,k = 0,1,\ldots,D-1$, $-\infty < y < +\infty$, $0 \leq z < \infty$, and η_{ik} is the Minkowskian metric tensor in D-dimensional subspace. The subspace $(x^1,\ldots,x^{D-1})$ has the topology $R^p \times T^q$,

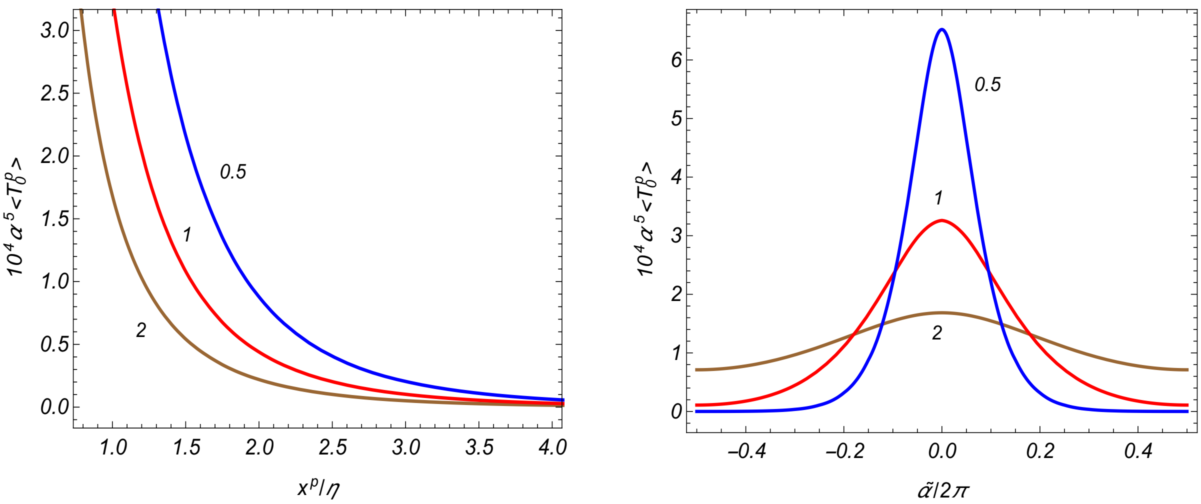

Figure 2: The energy flux density versus the distance from the brane (left panel), and as a function of the phase in the periodicity condition along compact dimension (right panel) for a conformally coupled scalar field with Dirichlet boundary condition.

$q = D - p - 1$, with the lengths of compact dimensions L_i, $i = p+1, \ldots, D-1$. Along those dimensions the field obeys quasiperiodicity conditions with phases α_l, similar to (3). The second sort of boundary condition is imposed by the presence of a brane parallel to the AdS boundary and located at $y = y_0$. We consider the bag boundary condition $(1 + i\gamma^\mu n_\mu)\psi(x) = 0$, with n_μ being the corresponding normal, that is the most popular condition used for the confinement of fermions in a variety of situations. In addition to the background gravitational field, we also assume the presence of a constant abelian gauge field A_μ. Yet another kind of boundary condition should be imposed on normalizable irregular modes at the timelike boundary of the AdS spacetime corresponding to $z = 0$. We consider a special case of allowed boundary conditions corresponding to the bag boundary condition on a hypersurface close to the AdS boundary with the subsequent limiting transition to the AdS boundary.

Because of the global nature of the vacuum in quantum field theory, the expectation values of local physical observables are sensitive to the boundary conditions on the field. As such a local observable we consider the fermionic current density $j^\mu = e\bar{\psi}\gamma^\mu\psi$. The vacuum expectation values for the charge density and for the components of the current along uncomapct dimensions vanish: $\langle j^\mu \rangle = 0$ for $\mu = 0, 1, \ldots, p$. The brane divides the space into two regions with different properties of the vacuum and we consider the vacuum expectation values for the components of the current along compact dimensions in these regions separately. Complete sets of solutions of the Dirac equation in both these regions are found. The dependence of the modes on the radial coordinate y is expressed in terms of the cylinder functions $Z_{ma\pm 1/2}(\lambda z)$, where m is the field mass. The vacuum expectation value of the current density is decomposed as $\langle j^l \rangle = \langle j^l \rangle_0 + \langle j^l \rangle_b$, where $\langle j^l \rangle_0$ is the current density in the geometry without the brane and $\langle j^l \rangle_b$ is induced by the brane. The contribution $\langle j^l \rangle_0$ was investigated in [6] and we are mainly concerned about that part $\langle j^l \rangle_b$.

In the region between the brane and AdS boundary, $0 \leq z \leq z_0$, $z = ae^{y_0/a}$, the eigenvalues of the quantum number λ are roots of the equation $J_{ma-1/2}(\lambda z_0) = 0$. In order to extract the contribution induced by the brane, we apply to the series over these eigenvalues in the mode-sum for the current density the Abel-Plana type summation formula. For the component along the lth compact dimension this contribution is given by

$$\langle j^l\rangle_b = -\frac{NeA_p z^{D+2}}{V_q a^{D+1}} \sum_{\mathbf{n}_q} k_l \int_{k_{(q)}}^{\infty} du\, u \left(u^2 - k_{(q)}^2\right)^{\frac{p-1}{2}} \frac{K_{ma-1/2}(uz_0)}{I_{ma-1/2}(uz_0)} \left[I^2_{ma+\frac{1}{2}}(uz) - I^2_{ma-\frac{1}{2}}(uz)\right], \tag{11}$$

where $N = 2^{[(D+1)/2]}$, $k_{(q)}^2 = \sum_{i=p+1}^{D-1} k_i^2$, $\mathbf{n}_q = (n_{p+1}, \ldots, n_{D-1})$, and

$$A_p = \frac{(4\pi)^{-(p+1)/2}}{\Gamma\left((p+1)/2\right)}, \quad k_i = \frac{2\pi n_i + \tilde{\alpha}_i}{L_i}, \quad n_i = 0, \pm 1, \pm 2, \ldots. \tag{12}$$

It is an odd periodic function of $\tilde{\alpha}_l$ and an even periodic function of the remaining phases $\tilde{\alpha}_i$, $i \neq l$, with the period equal to 2π. For a massless field the result on the locally AdS bulk is conformally related to the corresponding result on a locally Minkowski bulk in the region between two parallel planar boundaries. The latter are conformal images of the AdS boundary and of the brane. For a massive field, we have also checked the limiting transition to the corresponding result in the problem with a single boundary on a locally Minkowski spacetime, considered in [7]. On the AdS boundary, both the brane-free and brane-induced contributions to the current density tend to zero as $z^{D+2ma+1}$. An important feature that distinguishes the vacuum expectation value of the current density from the vacuum expectation value of the energy-momentum tensor is the finiteness of the former on the brane. For the investigation of the near-brane asymptotic we have provided an alternative representation. For $z \gg L_i$ and at distances from the brane corresponding to $z_0 - z \gg L_i$, the brane-induced contribution behaves as $\langle j^l\rangle_b \propto z^D e^{-2(z_0-z)k_{(q)}^{(0)}} / \left(z_0 - z\right)^{(p+1)/2}$, with $k_{(q)}^{(0)2} = \sum_{i=p+1}^{D-1} \tilde{\alpha}_i^2/L_i^2$, $|\tilde{\alpha}_i| < \pi$, and it is mainly localized near the brane in the region $z_0 - z \lesssim L_i$. For a fixed observation point, when the brane is close to the AdS horizon, the brane-induced current density is suppressed by the factor $e^{-2k_{(q)}^{(0)} z_0}$. If the length of the lth compact dimension is much smaller than the lengths of the remaining dimensions, the leading term in the expansion of the component $\langle j^l\rangle_b$ coincides with the current density in the model with a single compact dimension x^l when the remaining compact dimensions are decompactified. If, in addition, one has $L_l \ll z_0 - z$, then the brane-induced current density decays like $e^{-2(z_0-z)|\tilde{\alpha}_l|/L_l}$. This feature is seen in figure 3. In the opposite limit of large values for L_l, the brane-induced current density along the corresponding dimension is suppressed by the factor $\exp[-L_l\sqrt{\lambda_1^2/z_0^2 + k_{(q-1)}^{(0)2}}]$, where $k_{(q-1)}^{(0)2} = k_{(q)}^{(0)2} - \tilde{\alpha}_l^2/L_l^2$ and $\lambda = \lambda_1$ is the smallest positive root of the equation $J_{ma-1/2}(\lambda z_0) = 0$.

In the region between the brane and the AdS horizon, $z_0 \le z < \infty$, the eigenvalues for λ are continuous. The expression for the brane-induced contribution to the vacuum expectation value of the current density in this region is given by

$$\langle j^l\rangle_b = \frac{NeA_p z^{D+2}}{V_q a^{D+1}} \sum_{\mathbf{n}_q} k_l \int_{k_{(q)}}^{\infty} du\, u \left(u^2 - k_{(q)}^2\right)^{\frac{p-1}{2}} \frac{I_{ma+1/2}(uz_0)}{K_{ma+1/2}(uz_0)} \left[K^2_{ma+\frac{1}{2}}(uz) - K^2_{ma-\frac{1}{2}}(uz)\right]. \tag{13}$$

For a massless field, the problem is conformally related to the corresponding problem on a locally Minkowskian spacetime with a single boundary and the brane-induced contribution vanishes in the R-region. In the case of a massive field and at large distances from the brane ($z \gg z_0, L_i$, points near the AdS horizon), that contribution is suppressed by the factor $e^{-2zk_{(q)}^{(0)}}$. In this region the total current density is dominated by the brane-free part. For a fixed observation

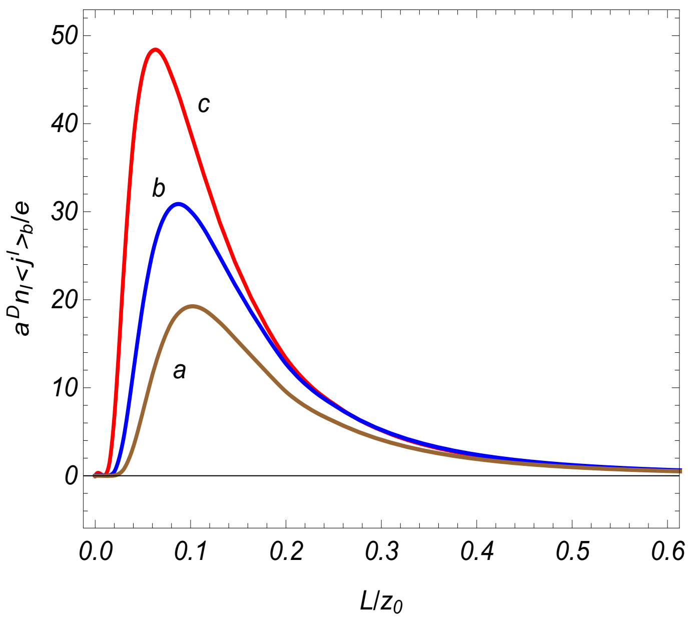

Figure 3: The brane-induced charge flux along the compact dimension, $a^D n_l^{(l)} \langle j^l \rangle$, as a function of the rescaled length L/z_0 for separate values of the phase $\tilde{\alpha} = 2\pi/3$ (a), $\tilde{\alpha} = \pi/2$ (b), $\tilde{\alpha} = \pi/3$ (c). For the mass we have taken $ma = 2$ and $z/z_0 = 0.95$.

point, when the brane location tends to the AdS boundary, $z_0 \to 0$, the brane-induced current density tends to zero as z_0^{2ma+1}. If the length L_l is much smaller than the lengths of the remaining compact dimensions and also $L_l \ll z - z_0$, similar to the case of the L-region, the brane-induced current decays as $e^{-2(z-z_0)|\tilde{\alpha}_l|/L_l}$. In the opposite limit of large values for L_l, the asymptotic behavior of the current is essentially different depending on the phases in the periodicity conditions along the remaining compact dimensions. If at least one of the phases $\tilde{\alpha}_i$, $|\tilde{\alpha}_i| < \pi$, $i \neq l$, is different from zero, the leading term in the asymptotic expansion has an exponential suppression like $e^{-L_l k_{(q-1)}^{(0)}}$. In the case $\tilde{\alpha}_i = 0$, the decay of the current density as a function of L_l is power law for both massive and massless fields.

In Z_2-symmetric braneworlds of the Randall-Sundrum type the geometry is composed by two copies of the R-region related by the Z_2-symmetry identification. Depending on the transformation of the field under the Z_2-reflection two types of the boundary conditions on the brane are obtained. The first one corresponds to the bag boundary condition. The current density in this case coincides with that investigated above with an additional factor 1/2, related to the presence of two copies of the R-region. For the second boundary condition the expression for the brane-induced vacuum expectation value of the current density is obtained from (11) by the replacements $I \rightleftarrows K$ of the modified Bessel functions with an additional coefficient $1/2$. Now, in the range of the mass $ma < 1/2$, the brane-induced current density does not vanish in the limit when the brane tends to the AdS boundary.

In odd-dimensional spacetimes the mass term for a field realizing the irreducible representation of the Clifford algebra breaks the invariance with respect two of the C-, P- and T-transformations. Models invariant under these transformations are constructed by combining the fields realizing two inequivalent representations of the Clifford algebra. We have shown that, if the phases in the quasiperiodicity conditions are the same for these fields, they give the same contribution to the total current density in this kind of models. From the point of view of applications of fermionic models in condensed matter physics, an important special case corresponds to three-dimensional spacetime ($D = 2$). An example of physical realization of those models is graphene. The background topology in this model is nontrivial for carbon nanotubes and nanoloops (cylindrical and toroidal topologies, respectively). The phase for the fermionic field along compact dimension of the nanotube depends on the chirality of the tube. For deformed tubes with z-dependent radius, the resulting current is obtained from general formulas taking $D = 2$ and $\alpha_D = 0$ for metallic tubes and $\alpha_D = \pm 2\pi/3$ for semiconducting tubes, respectively. For semiconducting tubes, the upper and lower signs stand for the spinors corresponding to the points $\mathbf{K}_+$ and $\mathbf{K}_-$ of the graphene Brillouin zone. As a consequence of the cancellation of the contributions from these spinors, the total current vanishes in the absence of the magnetic flux Φ threading the tube.

In **Chapter 4** the influence of a plate parallel to the AdS boundary on the properties of the electromagnetic field is studied. The corresponding line element has the form (10) and the plate is located at $z = z_0$. On it the field obeys the boundary condition $n^\mu F^*_{\mu\nu_1\ldots\nu_{D-1}} = 0$, where n^μ is the normal vector to the plate, $F^*_{\mu\nu_1\ldots\nu_{D-1}}$ is the dual of the field tensor $F_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$. For $D = 3$ this corresponds to the perfectly conducting boundary condition. The complete sets of the electromagnetic field mode functions are found in both the regions $0 \leq z \leq z_0$ and $z_0 \leq z < \infty$. In the region $0 \leq z \leq z_0$ the eigenvalues of the radial quantum number λ are solutions of the equation $J_{D/2-1}(\lambda z_0) = 0$. The two-point functions for the vector potential, $\langle A_\mu(x)A_\nu(x')\rangle$, and for the field tensor, $\langle F_{\sigma\mu}(x)F_{\rho\nu}(x)\rangle$, are decomposed into the boundary-free (the plate is absent) and plate-induced contributions. On the base of that, a similar decomposition is obtained for the vacuum expectation value of the energy-momentum tensor: $\langle T^\mu_\nu\rangle = \langle T^\mu_\nu\rangle_0 + \langle T^\mu_\nu\rangle_b$. For points outside the plate the renormalization is required for the boundary-free part $\langle T^\mu_\nu\rangle_0$ only. Because of the maximal symmetry of the background geometry, the latter is proportional to the metric tensor, $\langle T^\mu_\nu\rangle_0 = \mathrm{const} \cdot \delta^\mu_\nu$.

The boundary-induced contribution $\langle T^\mu_\nu\rangle_b$ is diagonal and the corresponding components in the region $z_0 < z < \infty$ are given by the expression (no summation over μ)

$$\langle T^\mu_\mu\rangle_b = -\frac{(D-1)(z/z_0)^{D+2}}{(4\pi)^{D/2}\Gamma(D/2+1)a^{D+1}} \int_0^\infty dx x^{D+1} \frac{I_{D/2-1}(x)}{K_{D/2-1}(x)} G^{(\mu)}_{D/2-1}(xz/z_0), \tag{14}$$

where $G^{(\mu)}_\nu(u) = \nu K^2_{\nu-1}(u) + (\nu-1)K^2_\nu(u)$ for $\mu = 0, 1, ..., D-1$, and $G^{(D)}_\nu(u) = (\nu+1)[K^2_\nu(u) - K^2_{\nu-1}(u)]$. The expression for $\langle T^\mu_\mu\rangle_b$ in the region $0 < z < z_0$ is obtained from (14) by the replacements $I_\nu \rightleftarrows K_\nu$. As seen from (14), the vacuum stresses along the directions parallel to the plate, $\langle T^\mu_\mu\rangle$, $\mu = 1, ..., D-1$, are equal to the energy density. This result could also be directly obtained on the base of the Lorentz invariance of the problem with respect to the boosts along the directions parallel to the plate. In the region $z_0 < z < \infty$, the boundary-induced contribution vanishes for $D = 3$. We could expect this result from the conformal relation to the corresponding problem in Minkowski bulk with a single plate. In the region $0 < z < z_0$, the plate-induced contribution does not vanish for $D = 3$. This is related to the fact that in that region the problem with $D = 3$ is conformally related to the problem in the Minkowski bulk with two plates, where the second one corresponds to the AdS boundary $z = 0$. For $D \geq 4$, we have $\langle T^l_l\rangle_b < 0$ for $l = 0, 1, ..., D-1$, in both the regions on the right and on the left of the plate, whereas $\langle T^D_D\rangle_b < 0$ for $z_0 < z < \infty$ and $\langle T^D_D\rangle_b > 0$ for $0 < z < z_0$.

The boundary-induced vacuum expectation value (14) depends on z and z_0 in the form of the ratio z/z_0 which is related to the proper distance from the plate, $y - y_0$, by $z/z_0 = e^{(y-y_0)/a}$. Hence, for a given proper distance, the vacuum expectation values do not depend on the location of the boundary. The latter property is a consequence of the maximal symmetry of the AdS spacetime. It can be seen that the boundary-induced contributions in the vacuum expectation value of the energy-momentum tensor obey the continuity equation $\nabla_\mu \langle T_\nu^\mu \rangle_b = 0$ which for the geometry under consideration takes the form $z^{D+1}\partial_z(z^{-D}\langle T_D^D \rangle_b) + D\langle T_0^0 \rangle_b = 0$.

At distances from the plate larger than the AdS curvature radius one has $y - y_0 \gg a$ and, hence, $z/z_0 \gg 1$. To the leading order, $\langle T_0^0 \rangle_b \approx 2(1 - 1/D)\langle T_D^D \rangle_b \propto (z/z_0)^{2-D}$ and the plate-induced parts are suppressed as functions of the proper distance by the factor $e^{-(D-2)(y-y_0)/a}$. At small distances from the plate, compared with the AdS curvature radius, one has $|y - y_0| \ll a$ and, hence, $|1 - z_0/z| \ll 1$. In this case, to the leading order we get $\langle T_D^D \rangle_b \approx (y - y_0)\langle T_0^0 \rangle_b/a$ and

$$\langle T_0^0 \rangle_b \approx -\frac{(D-1)(D-3)\Gamma((D+1)/2)}{2(4\pi)^{(D+1)/2}|y - y_0|^{D+1}}.$$

The leading term for the energy density does not depend on the curvature radius a and coincides with the corresponding result for the plate in Minkowski spacetime. Hence, in this region the effect of gravity on the boundary-induced energy density is small.

CONCLUSIONS

1. The influence of compactification of spatial dimensions on vacuum fluctuations for a charged scalar field in locally dS spacetime is investigated. For lengths of the compact dimensions smaller than the curvature radius of the space the effect of the gravitational field on the vacuum expectation values of the field squared and energy-momentum tensor is weak. In the early stages of the cosmological expansion the topological parts dominate in the expectation values.

2. Depending on the mass and on the curvature coupling parameter of the field, at late stages of the expansion the topological contributions in the expectation values decay monotonically or damping oscillatory. Unlike to the locally Minkowskian background geometry with compact dimensions, for dS spacetime the vacuum effective pressure along uncompact dimensions is not equal to the energy density and the vacuum energy-momentum tensor, as a source of the gravitational field, is not of the cosmological constant type.

3. The vacuum expectation values of the field squared and energy-momentum tensor induced by a planar boundary in dS spacetime with compact dimensions are studied. Near the boundary the main contribution to the expectation values comes from the vacuum fluctuations with small wavelengths and the influence of the gravitational field is small. The effect of the gravity on the boundary-induced expectation values in the energy density and pressures is essential at distances from the boundary larger than the de Sitter curvature radius. Depending on the values of the parameters the vacuum energy density can be either positive or negative.

4. In addition to diagonal components of the vacuum energy-momentum tensor, the presence of a boundary induces an off-diagonal component that describes energy flux along the normal to the boundary. The flux can be either positive or negative, corresponding to the flux direction from or to the boundary. For a conformally coupled massless field the energy flux is absent.

5. The influence of a brane on the vacuum expectation value of the current density is investigated for a charged fermionic field in background of locally AdS spacetime with an arbitrary number of toroidally compact dimensions and in the presence of a constant gauge field. The vacuum expectation values for the charge density and the components of the current density along uncompact dimensions vanish. The brane-induced contributions in the components along compact dimensions are mainly located near the brane and vanish on the AdS boundary and on the horizon. Applications are given to Z_2-symmetric braneworlds of the Randall-Sundrum type with compact dimensions for two classes of boundary conditions on the fermionic field. In these models the vacuum currents along compact dimension are sources of magnetic fields in the uncompact subspace.

6. In odd spacetime dimensions, the fermionic fields realizing two inequivalent irreducible representations of the Clifford algebra and having equal phases in the periodicity conditions give the same contribution to the vacuum current density. Combining the contributions from these fields, the current density in odd-dimensional C-,P- and T-symmetric models is obtained. In the special case of three-dimensional spacetime, the corresponding results are applied for the investigation of the edge effects on the ground state current density induced in curved graphene tubes by an enclosed magnetic flux.

7. The electromagnetic field two-point functions and the vacuum expectation value of the energy-momentum tensor are investigated in AdS spacetime in the presence of a plate parallel to the AdS horizon. For spatial dimension larger than three, the boundary-induced vacuum energy density is negative and the normal stress corresponds to positive pressure in the region between the plate and the AdS horizon and the negative pressure in the region between the plate and the AdS boundary. At distances from the plate larger than the AdS curvature radius the effect of gravity is crucial and the boundary-induced contributions are exponentially suppressed. For points near the plate the contribution of the vacuum fluctuations with the wavelengths much smaller than the AdS curvature radius dominates and the boundary-induced vacuum expectation values, in the leading order, coincide with the corresponding vacuum expectation values for a plate in Minkowski bulk.

REFERENCES

1. M. Bordag, G.L. Klimchitskaya, U. Mohideen, and V.M. Mostepanenko, *Advances in the Casimir Effect* (Oxford University Press, Oxford, 2009).

2. *Lecture Notes in Physics: Casimir Physics,* Vol. 834, edited by D. Dalvit, P. Milonni, D. Roberts, and F. da Rosa (Springer, Berlin, 2011).

3. A. Linde, Creation of a compact topologically nontrivial inflationary universe. J. Cosmol. Astropart. Phys. JCAP10(2004)004.

4. X.-L. Qi, S.-C. Zhang, Topological insulators and superconductors. Rev. Mod. Phys. **83**, 1057 (2011).

5. A. Furusaki, N. Nagaosa, K. Nomura, S. Ryu, T. Takayanagi, Electromagnetic and thermal responses in topological matter: Topological terms, quantum anomalies and D-branes. C. R. Physique **14**, 871 (2013).

6. S. Bellucci, A.A. Saharian, V. Vardanyan, Fermionic currents in AdS spacetime with compact dimensions. Phys. Rev. D **96**, 065025 (2017).

7. S. Bellucci, A.A. Saharian, Fermionic current from topology and boundaries with applications to higher-dimensional models and nanophysics. Phys. Rev. D **87**, 025005 (2013).

PUBLICATIONS IN THE TOPIC OF THESIS

1. A.S. Kotanjyan, A.A. Saharyan, H.G. Sargsyan, D.H. Simonyan, Quantum effects in anti-de Sitter spacetime for the electromagnetic vacuum. Proceedings of Science PoS(MPCS2015) 021 (2015), 13 pages.

2. E.V. Chubaryan, A.S. Kotanjyan, A.A. Saharian, D.H. Simonyan, Vacuum densities for a charged scalar field in de Sitter spacetime with compact dimensions. Gravitation and Cosmology **22**, 2, 187-192 (2016).

3. S. Bellucci, A.A. Saharian, D.H. Simonyan, V.V. Vardanyan, Fermionic currents in topologically nontrivial braneworlds. Physical Review D **98**, 085020 (2018), 22 pages.

4. D.H. Simonyan, Vacuum fluctuations for a charged scalar field in de Sitter spacetime with compact dimensions. Armenian Journal of Physics **11** (3), 145-152 (2018).

5. A.A. Saharian, D.H. Simonyan, A.S. Kotanjyan, Vacuum polarization induced by a boundary in de Sitter space with compact dimensions. Journal of Contemporary Physics **54** (1), 1-10 (2019).

6. A.A. Saharian, D.H. Simonyan, T.A. Petrosyan, Vacuum fluxes from a brane in de Sitter spacetime with compact dimensions. Armenian Journal of Physics **12** (1), 51-61 (2019).

ԱՄՓՈՓԱԳԻՐ

Ատենախոսությունում հետազոտված են քվանտային վակուումի հատկությունները ոչ-տրիվիալ տոպոլոգիայով դե Սիտտերի ու անտի-դե Սիտտերի տարածություններում, երբ առկա են նաև սահմաններ: Որպես վակուումի կարևոր լոկալ բնութագրեր, դիտարկված են դաշտի քառակուսու և էներգիա-իմպուլսի թենզորի վակուումային միջինները սկալյար ու էլեկտրամագնիսական դաշտերի համար և հոսանքի խտության վակուումային միջինը լիցքավորված ֆերմիոնային դաշտի համար: Վակուումային միջիններում բացահայտ կերպով առանձնացված են տոպոլոգիական ու սահմաններով մակածված մասերը և հետազոտված է դրանց վարքը պարամետրերի արժեքների տարբեր սահմանային տիրույթներում: Քննարկված են կիրառությունները կոսմոլոգիական մոդելներում, բրան աշխարհների մոդելներում և գրաֆենային նանոխողովակներում:

1. Հետազոտված է տարածական չափողականությունների կոմպակտիֆիկացիայի ազդեցությունը լոկալ դե Սիտտերի տարածաժամանակում լիցքավորված սկալյար դաշտի վակուումային ֆլուկտուացիաների վրա: Կոմպակտ չափերի՝ տարածության կորության շառավղից փոքր երկարությունների համար, գրավիտացիոն դաշտի ազդեցությունը դաշտի քառակուսու և էներգիա-իմպուլսի թենզորի վակուումային միջինների վրա թույլ է: Կոսմոլոգիական ընդարձակման վաղ փուլերում տոպոլոգիական ներդրումները գերակշռում են վակուումային միջիններում:

2. Կախված դաշտի զանգվածից և կորության կապի պարամետրից՝ ընդարձակման ուշ փուլերում վակուումային միջիններում տոպոլոգիական ներդրումները մարում են մոնոտոն կամ օսցիլյացիոն կերպով: Ի տարբերություն կոմպակտ չափողականություններով լոկալ Մինկովսկու ֆոնային երկրաչափության՝ դե Սիտտերի տարածաժամանակի համար ոչ կոմպակտ չափողականությունների երկայնքով վակուումային էֆեկտիվ ճնշումը հավասար չէ էներգիայի խտությանը, և վակուումային էներգիա-իմպուլսի թենզորը, որպես գրավիտացիոն դաշտի աղբյուր, կոսմոլոգիական հաստատունի տիպի չէ:

3. Ուսումնասիրված են դաշտի քառակուսու և էներգիա-իմպուլսի թենզորի՝ տարածական սահմանով մակածված վակուումային միջինները կոմպակտ չափողականություններով դե Սիտտերի տարածաժամանակում: Սահմանին մոտ վակուումային միջինների հիմնական ներդրումը պայմանավորված է փոքր ալիքի երկարությամբ վակուումային ֆլուկտուացիաներով և գրավիտացիոն դաշտի ազդեցությունը փոքր է: Գրավիտացիայի ազդեցությունը էներգիայի խտության և ճնշումների՝ սահմանով մակածված միջինների վրա էական է սահմանից՝ դե Սիտտերի կորության շառավղից մեծ հեռավորությունների վրա: Կախված պարամետրերի արժեքներից՝ վակուումային էներգիայի խտությունը կարող է լինել դրական կամ բացասական:

4. Վակուումային էներգիա-իմպուլսի թենզորի անկյունագծային բաղադրիչներից բացի, սահմանի առկայությունը մակածում է ոչ անկյունագծային բաղադրիչ, որը նկարագրում է սահմանին ուղղահայաց էներգիայի հոսք: Հոսքը կարող է լինել դրական կամ բացասական՝ համապատասխանելով սահմանից կամ դեպի սահման հոսքի ուղղություններին: Կոնֆորմ կապված զրոյական զանգվածով դաշտի համար էներգիայի հոսքը բացակայում է:

5. Հետազոտված է բրանի ազդեցությունը հոսանքի խտության վակուումային միջինի վրա կամայական թվով տորոիդալ կոմպակտիֆիկացված չափողականություններով լոկալ անտի-դե Սիտտերի տարածաժամանակում՝ հաստատուն տրամաչափային դաշտի առկայությամբ: Լիցքի խտության և հոսանքի խտության՝ ոչ կոմպակտ չափողականությունների երկայնքով բաղադրիչների վակուումային միջինները զրոյանում են: Կոմպակտ չափողականությունների երկայնքով բաղադրիչներում բրանով մակածված ներդրումները հիմնականում տեղայնացված են բրանի շրջակայքում և զրոյանում են անտի-դե Սիտտերի սահմանի և հորիզոնի վրա: Բերված են կիրառություններ կոմպակտ չափողականություններով Ռանդալ-Սունդրումի տիպի Z_2-համաչափ բրան աշխարհներում ֆերմիոնային դաշտի վրա երկու տիպի եզրային պայմանների համար: Այս մոդելներում կոմպակտ չափողականության երկայնքով վակուումային հոսանքները հանդիսանում են ոչ կոմպակտ ենթատարածությունում մագնիսական դաշտերի աղբյուր:

6. Տարածաժամանակի կենտ չափողականությունների համար, ֆերմիոնային դաշտերը՝ իրականացնելով Քլիֆորդի հանրահաշվի երկու անհամարժեք չբերվող ներկայացումներ և պարբերականության պայմաններում ունենալով հավասար փուլեր, վակուումային հոսանքի խտությունում տալիս են նույն ներդրումը: Միավորելով այս դաշտերի ներդրումները՝ ստացված է կենտ չափողականությամբ $C-$, $P-$ և $T-$համաչափ մոդելներում հոսանքի խտությունը: Եռաչափ տարածաժամանակի մասնավոր դեպքում համապատասխան արդյունքները կիրառված են կորացած գրաֆենային խողովակներում՝ հիմնական վիճակում մագնիսական հոսքով մակածված հոսանքի խտության վրա եզրերի ազդեցությունը հետազոտելու համար:

7. Հետազոտված են էլեկտրամագնիսական դաշտի երկկետային ֆունկցիաները և էներգիա-իմպուլսի թենզորի վակուումային միջինը անտի-դե Սիտտերի տարածաժամանակում՝ վերջինիս հորիզոնին զուգահեռ սահմանի առկայությամբ: Տարածության չափողականության երեքից մեծ արժեքների համար սահմանով մակածված վակուումային էներգիայի խտությունը բացասական է, իսկ նորմալ լարվածությունը համապատասխանում է դրական ճնշման սահմանի ու հորիզոնի միջև տիրույթում և բացասական ճնշման՝ սահմանի ու անտի-դե Սիտտերի տարածության եզրի միջև տիրույթում: Սահմանից՝ անտի-դե Սիտտերի կորության շառավղից մեծ հեռավորությունների վրա գրավիտացիայի ազդեցությունը էական է, իսկ սահմանով մակածված ներդրումներն էքսպոնենցիալ փոքր են: Սահմանին մոտ կետերի համար գերակշռում է անտի-դե Սիտտերի կորության շառավղից փոքր ալիքի երկարությամբ վակուումային ֆլուկտուացիաների ներդրումը, և սահմանով մակածված վակուումային միջինները, առաջին մոտավորությամբ, համընկնում են Մինկովսկու երկրաչափությունում թիթեղի համար համապատասխան վակուումային միջինների հետ:

СИМОНЯН ДАВИД
ВАКУУМНЫЕ КВАНТОВЫЕ ЭФФЕКТЫ В ИСКРИВЛЕННЫХ ПРОСТРАНСТВАХ, ИНДУЦИРОВАННЫЕ ТОПОЛОГИЕЙ

В диссертации исследованы свойства квантового вакуума в пространствах де Ситтера и анти-де Ситтера с нетривиальной топологией при наличии границ. В качестве важных локальных характеристик вакуумного состояния рассмотрены вакуумные средние квадрата поля и тензора энергии-импульса для скалярного и электромагнитного полей и вакуумное среднее плотности тока для заряженного фермионного поля. Явно выделены части вакуумных средних, индуцированные топологией и границами и исследовано их поведение в различных граничных областях значений параметров. Обсуждаются приложения в космологических моделях, в моделях бран миров и в графеновых нанотрубках.

1. Исследовано влияние компактификации пространственных измерений на вакуумные флуктуации для заряженного скалярного поля в локальном пространстве-времени де Ситтера. Для длин компактных измерений, меньших радиуса кривизны пространства, влияние гравитационного поля на вакуумные средние квадрата поля и тензора энергии-импульса слабое. На ранних стадиях космологического расширения топологические части доминируют в вакуумных средних.

2. В зависимости от массы и параметра связи кривизны поля, на поздних стадиях расширения топологические вклады в вакуумных средних затухают монотонно или осцилляционно. В отличие от локально-минковской фоновой геометрии с компактными измерениями, для пространства-времени де Ситтера эффективное давление вакуума вдоль некомпактных измерений не равно плотности энергии, а тензор энергии-импульса вакуума, как источник гравитационного поля, не является источником типа космологической постоянной.

3. Изучены вакуумные средние квадрата поля и тензора энергии-импульса, индуцированные плоской границей в пространстве-времени де Ситтера с компактными измерениями. Вблизи границы основной вклад в средние вносят вакуумные флуктуации с малыми длинами волн и влияние гравитационного поля мало. Влияние гравитации на индуцированные границей плотность энергии и давления существенно на расстояниях от границы, больших по сравнению с радиусом кривизны пространства де Ситтера. В зависимости от значений параметров, вакуумная плотность энергии может быть как положительной, так и отрицательной.

4. В дополнение к диагональным компонентам вакуумного тензора энергии-импульса, наличие границы индуцирует ненулевую недиагональную компоненту, которая описывает поток энергии вдоль нормали к границе. Поток может быть положительным или отрицательным, что соответствует направлению потока от границы или к границе. Для конформно связанного безмассового поля поток энергии отсутствует.

5. Исследовано влияние браны на вакуумное среднее плотности тока для заряженного фермионного поля на фоне локального пространства-времени анти-де Ситтера

с произвольным числом тороидально компактных измерений и при наличии постоянного калибровочного поля. Вакуумные средние плотности заряда и компонент плотности тока вдоль некомпактных измерений равны нулю. Индуцированные браной вклады в компонентах вдоль компактных измерений в основном локализованы вблизи браны и исчезают на границе анти-де Ситтера и на горизонте. Даны приложения к Z_2-симметричным бран-мирам типа Рэндалла-Сундрума с компактными измерениями для двух классов граничных условий на фермионное поле. В этих моделях вакуумные токи вдоль компактных измерений являются источником магнитных полей в некомпактном подпространстве.

6. В нечетных пространственно-временных измерениях фермионные поля, реализующие два неэквивалентных неприводимых представления алгебры Клиффорда и имеющие равные фазы в условиях периодичности, дают одинаковый вклад в вакуумную плотность тока. Суммированием вкладов этих полей получена плотность тока в нечетномерных $C-$, $P-$ и $T-$симметричных моделях. В частном случае трехмерного пространства-времени соответствующие результаты применяются для исследования краевых эффектов на плотность тока основного состояния, индуцированную приложенным магнитным потоком в деформированных графеновых трубках.

7. Исследовались двухточечные функции электромагнитного поля и вакуумное среднее тензора энергии-импульса в пространстве-времени анти-де Ситтера при наличии границы, параллельной горизонту анти-де Ситтера. Для пространственного измерения больше трех, индуцированная границей вакуумная плотность энергии отрицательна, а нормальное напряжение соответствует положительному давлению в области между границей и горизонтом и отрицательному давлению в области между рассматриваемой границей и границей пространства анти-де Ситтера. На расстояниях от границы, больших по сравнению с радиусом кривизны анти-де Ситтера, влияние гравитации является существенным и обусловленные границей вклады экспоненциально малы. Для точек вблизи границы доминирует вклад вакуумных флуктуаций с длинами волн, значительно меньших радиуса кривизны анти-де Ситтера и индуцированные границей вакуумные средние в ведущем порядке совпадают с соответствующими средними для границы в геометрии Минковского.