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SOME INITIAL –BOUNDARY AND VARIATIONAL PROBLEMS FOR PZEUDO-EVOLUTION EQUATIONS

SYNOPSIS

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Ատենախոսության թեման հաստատվել է ԵՊՀ մաթեմատիկայի և մեխանիկայի ֆակուլտետի խորհրդի կողմից:

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The thesis can be found in the library of the YSU.
Synopsis was sent on april 5, 2013.
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General characteristics of the work

Relevance of the theme.

In classic theory of partial differential equations, like Cauchy problem as well as various mixed problems, are being studied the kind of equations or equations or equation systems that are solved towards time high quality derivatives.

The study of many applied problems are brought to Cauchy or to the mixed problems that are not solved towards time high quality derivative. Such kind of problems were first studied by S.L. Sobolev and were connected to the special issues of ideal liguid hydrodynamic movement [1]. S.L. Sobolev not only gave the absolute solution of those problems but also discovered new qualitative properties that did not exist in classic systems.

In 1949 R.A. Aleksandrian [2] article it is showen that these qualitative properties (the behavior of solutions for enough great t-s in particular) are expressed more strongly if, not Cauchy problem, but a kind of problem will be considered where the domain of special variables has boundary. For the first time the following differential equation system with variable coefficients was being studied in some works of R.A. Aleksandrian [2] – [6].

$$\frac{\partial \vec{v}}{\partial t} = A(x)\ \vec{v} + B(x)\ grad\ P \qquad (1)$$

$$Div\ \vec{v} = 0 \qquad (2)$$

Where special variables $x = (x_1, x_2, x_3, \ldots, x_n)$ are being converted by enough smooth boundary in Ω limited domain, $t > 0,\ \vec{v} = \vec{v}(t, x)$ is in n-size vector function $p = p(t, x)$

In scalar function, $A(x),\ B(x)$ are enough smooth matrices. S.L. Sobolev problem, for the above mentioned system, is to fined such $\vec{v}(t, x)$ and $p(t, x)$ functions so that they satisfy

$$\vec{v}\ |_{t=0} = \vec{v}_0(x) \qquad (3)$$
$$p(t,x)|_{\partial\Omega} = 0$$

conditions.

These problems are different from classic problems in the thing that initial conditions are put on one class of functions, and boundary conditions on the other class. Though these problems are not classic, it was shown by R.A. Aleksandrian that they have solution and only one. It was shown [5] that this problem in equal to Cauchy problem for operator equation in corresponding functional space.

Cauchy problem for the following S.L. Sobolev type equations

$$M\left(t,\frac{1}{t}\frac{\partial}{\partial x_k}\right)\frac{\partial \vec{u}}{\partial t} = L\left(t,\frac{1}{t}\frac{\partial}{\partial x_k}\right)\vec{u}$$
$$\vec{u}\left(t_0,x_k\right) = \vec{\varphi}\left(x_k\right) \qquad (4)$$

Where M and L are polynomial $m \times m$ matrices, are studied in details in the works of S.A. Galpern [7] , [8]. In these work, in case of some limitations, the fundamental solution was built and formulas were carried out, that give solution to Cauchy problem.

The mixed problems for S.A. Sobolev type equations were also studied in the works of A.G. Kostuchenko and G.I.Eskin [9], M.I. Vishik [10], Zelenyak [11] – [12], G.V. Virabyan [13], V.N. Maslenikova [14] – [16], S.V. Uspenskil [17,18], Showalter R.E. [19], Showalter R.E., Ting T.W. [20], Lagnese J.E. [21], Showalter R.E [22]-[23],Ting T.W.[24],Gaevski Kh, Groger K, Zakharias. K. [25], Hakobyan G. S. [26], Hakobyan G. S., Shakhbagyan R. L [27]-[31].

The aim of the thesis.

In this thesis initial-boundary value problem for a class of second order degenerate pseudoevolution equation with corresponding variational inequalities is considered.

- Study a class of pseudohyperbolic equation of second order with degeneration
- Study a class of boundary value problem for the pseudoparabolic equations for the case of non-standard initial vales
- Formulation of the free boundary problem, reduction to the equivalent variational inequality and proof of existence, uniqueness and smoothness theroems for the weak solutions

Scientific innovation. All results are new.

Practical and theoretical value. The results of the work have theoretical character. The results of the work can be used in the study of partial differential equaltions and variational inequalities.

Approbation of the results. The obtained results were presented

- at the research seminars of the chair of Differentail Equations of the Yerevan State University

- at the international conferance of Harmonic Analysis and Applications, 10-17 September 2011, Tsaghkadzor, Armenia

The main results of the thesis

The thesis consists of the introduction, two chapters and bibliography.
Chapter 1 studies some problems for pseudo– evolution equations.
In section 1.1 the statement of problem is shown and the main functional spaces are structured where the problem should be studied.
Section 1.2 studies the following below problem.
Let Ω be a bounded domain in $n-$ dimensional vector space R^n located in half-space $x_n>0$. We suppose that the boundary of the domain has the form $\partial\Omega=\Gamma_1\cup\Gamma_0$, where $\Gamma_0=\partial\Omega\cap\{x_n=0\}$ is a domain in the hyper plane $\{x_n=0\}$, $\Gamma_1=\partial\Omega\Gamma_0$ and for the domain Ω are valid the Sobolev embedding theorems.

In the cylinder $Q=\Omega\times R^+$ we consider the following initial – boundary value problem for the degenerate pseudohyperbolic equation

$$L\left(\frac{\partial^2 u}{\partial t^2}\right)+M(u)=0, \tag{5}$$

$$u|_{t=0}=u^{(0)}(x),\quad u_t|_{t=0}=u^{(1)}(x), \tag{6}$$

$$u|_{\Gamma^0}=0,\quad t>0, \tag{7}$$

Where

$$L(u)=-\sum_{i,j=1}^{n-1}\frac{\partial}{\partial x_i}\left(b_{ij}(x,t)\frac{\partial u}{\partial x_j}\right)-\frac{\partial}{\partial x_n}\left(b_{nn}(x)\frac{\partial u}{\partial x_n}\right),$$

$$M(u)=-\sum_{i,j=1}^{n-1}\frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_j}\right)-\frac{\partial}{\partial x_n}\left(a_{nn}(x,t)\frac{\partial u}{\partial x_n}\right).$$

It is assumed that the coefficients which appear in the definitions of the operators L and M satisfy to the conditions $a_{ij}(x,t)=a_{ij}(x,t)$, $b_{ij}(x,t)=b_{ij}(x,t)$

$(i,j=1,2,\dots,n)$ and are continuous and in $\bar{\Omega}\times R^{+}$, continuously differentiable with respect to the $x_1, x_2, x_3, \dots, x_n$ in $Q=\Omega\times R^{+}$, there are such exponents $\alpha>\beta\geq 0$, such that the products $x_n^{-\beta} b_{nn}(x,t)$, $|x_n^{-\alpha} a_{nn}(x,t)|$ are bounded from above and from below with positive constants, for every point $x\in\bar{\Omega}$ and for every $t\geq 0$ the quadratic form

$$\Lambda_0\left(x,t,\hat{\xi}\right)=\sum_{i,j=1}^{n-1} b_{ij}(x,t)\xi_i\xi_j$$

Is positive – definite, where $\hat{\xi}=\left(\xi_1,\dots,\xi_{n-1}\right)\in R^{n-1}$.

Γ^{*} is a part of the boundary, which, depending on the order of degeneracy β represents either the whole boundary $\partial\Omega$ or coincides with the Γ_1.

In the space $L_2(\Omega)$ define the operator L_β with the domain of definition $C_0^{\infty}(\Omega)$ by the formula

$$L_\beta(u)=-\sum_{i,j=1}^{n-1}\frac{\partial^2 u}{\partial x_i^2}-\frac{\partial}{\partial x_n}\left(x_n^{\beta}(x)\frac{\partial u}{\partial x_n}\right).$$

It follows from the results of the article [56], that the operator L_β is symmetric and positive – definite. Define a Hilbert space H_{L_β} as completion of the linear manifold $C_0^{\infty}(\Omega)$ in the metric generated by the following scalar product

$$(u,v)_{L_\beta}=\left(L_\beta(u),v\right)_0=\int_\Omega\left[\sum_{i=1}^{n-1}\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_i}+x_n^{\beta}(x)\frac{\partial u}{\partial x_n}\frac{\partial v}{\partial x_n}\right]dx$$

$T>0$, $Q=\Omega\times(0,T)$ is a cylinder with the base Ω, $\sum=\Gamma\times(0,T)$ is side of the cylinder Q.

Theorem 1. For any initial values $u^{(0)} \in H_{L_\beta}$ and $u^{(1)} \in H_{L_\beta}$ exists unique generalized solution of the problem (5) – (7) in H_{L_β}. For the case $\beta < 1$ we have $\Gamma^* = \partial\Omega$ and for $\beta \geq 1$ $\Gamma^* = \Gamma_1 = \partial\Omega \backslash \Gamma_0$.

In 1.3 the following problem has been studied.

Let $\Omega \subset \Re^n$ is a bounded domain with the smooth boundary Γ. We consider the following boundary value problem

$$\frac{\partial}{\partial t} L(u(t,x)) + M(u(t,x)) = f(t,x), \quad t > 0,\ x = (x_1, \ldots, x_n) \in \Omega \subset \Re^n \qquad (8)$$

$$u|_{\partial\Omega} = 0 \qquad (9)$$

$$(Lu)(0,x) = g(x), \quad x \in \Omega, \qquad (10)$$

Where

$$L(u) = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(b_{ij}(x,t) \frac{\partial u}{\partial x_j} \right),\ M(u) = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x,t) \frac{\partial u}{\partial x_j} \right),$$

$$f(t,x) \in L_2\left((0,T); W_2^{-1}(\Omega)\right),\ g(x) \in W_2^{-1}(\Omega).$$

We suppose that the functions $b_{ij}(t,x)$ and $a_{ij}(t,x)$ $(i,j = 1,2,\ldots,n)$ are defined in $[0,T] \times \overline{\Omega}$, $b_{ij}(t,x) = b_{ji}(t,x)$, $a_{ij}(t,x) = a_{ji}(t,x)$ $(i,j = 1,2,\cdots,n)$ and for every $t \in [0,T]$ and $x \in \overline{\Omega}$ the following quadratic form is positive defined

$$\sum_{i,j=1}^{n} b_{ij}(t,x) \xi_i \xi_j \geq c_0 |\xi|^2, \qquad (11)$$

where $\xi = (\xi_1, \ldots, \xi_n)$, $c_0 = const > 0$.

Theorem 2. Let the functions

$b_{ij}(t,x)=b_{ji}(t,x),\ a_{ij}(t,x)=a_{ji}(t,x)\quad (i=1,2,\ldots,n)$ are continuous in the domain $[0,T]\times\Omega$, for every $t\in[0,T]$ and every $x\in\bar{\Omega}$ is fulfilled the condition (11). Then the problem (8 – 10) has unique solution and

$$L(u)\in C\left(0,T;W_2^1(\Omega)\right),\ \frac{\partial}{\partial t}(L(u))\in L_2\left(0,T;W_2^1(\Omega)\right).$$

If L and M are second order nonlinear differential operators

$$L(u)=-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left(b_i(t,x,\nabla u)\right),\qquad M(u)=-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left(a_i(t,x,\nabla u)\right),$$

Where the functions $b_i(t,x,\xi_1,\ldots,\xi_n),\ a_i(t,x,\xi_1,\ldots,\xi_n)$ are defined and continuous in $[0,T]\times\bar{\Omega}\times R^n$ and have continuous derivatives with respect to $\xi_j\ (j=1,2,\ldots,n)$.

We suppose that the functions $b_i(t,x,\xi)$ and $a_i(t,x,\xi)$ $(\xi=(\xi_1,\ldots,\xi_n))(i=1,2,\ldots,n)$ fulfill the conditions

1. $|b_i(t,x,\xi)|\le c_1(|\xi|+1),\quad c_1=const>0,\ i=1,2,\ldots,n,$
2. $|b_{ij}(t,x,\xi)|=\left|\dfrac{\partial b_1}{\partial \xi_j}\right|\le c_2,\quad c_2=const>0,\ i,j=1,2,\ldots,n,$
3. $\sum_{i,j=1}^{n} b_{ij}(t,x,\xi)\eta_1\eta_2\ge c_3|\eta|^2\quad \forall t\in[0,T],\ \forall x\in\bar{\Omega}$

   $\forall\eta=(\eta_1,\ldots,\eta_n)\in R^n,$
4. $|a_i(t,x,\xi)|\le c_4(|\xi|+1),\quad \left|\dfrac{\partial a_i(t,x,\xi)}{\partial \xi_j}\right|\le c_5,\quad (i,j=1,2,\ldots,n).$

For fixed $t\in[0,T]$ define the operators $L(t)$ and $M(t)$ from $\dot{W}_2^1(\Omega)$ to $W_2^{-1}(\Omega)$ by the formulas

$$\langle L(t)v,w\rangle=\sum_{i=1}^{n}\int_{\Omega} b_i(t,x,\nabla u)\frac{\partial w}{\partial x_i}dx,$$

$$\langle M(t)v,w\rangle=\sum_{i=1}^{n}\int_{\Omega} a_i(t,x,\nabla u)\frac{\partial w}{\partial x_i}dx.$$

The operators $L(t)$ and $M(t)$ $(t\in[0,T])$ generate the mappings L and M from $L_2\left(0,T;\dot{W}_2^1(\Omega)\right)$ to $L_2\left(0,T;W_2^1(\Omega)\right)$ by the formulas

$$(Lu)(t)=L(t)\big(u(t,x)\big), \tag{12}$$

$$(Mu)(t)=M(t)\big(u(t,x)\big). \tag{13}$$

Lemma 1: Let the conditions (1-3) are fulfilled. Then the operator $L(t)$ is radial continuous and strong monotone.

Theorem 3: Let functions $b_i(t,x,\xi)$ and $a_i(t,x,\xi)$ $(i=1,2,\ldots,n)$ fulfill the conditions (1-4). Then the problem (8 – 10) where the operators L and M are defined by the formulae (12) and (13), has unique solution and $L(u)\in C\left(0,T;W_2^{-1}(\Omega)\right)$,

$$\frac{\partial}{\partial t}L(u)\in L_2\left(0,T;W_2^{-1}(\Omega)\right).$$

In chapter 2 we consider a free boundary problem of pseudopabolic type. To prove the existence of its solution, include the corresponding problem of variational inequalities. We prove that all solutions of the primary problem (i.e. free boundary problem) are solutions for the variational inequality and for sufficiently smooth solutions of the variational inequality is true the converse statement. So, the solution of variational

Inequalities are generalized solutions of a free boundary problem. Using the theory of pseudomonotone operators and linear semigroups, we prove theorems of existence, uniqueness and smoothness of weak solutions of variational inequalities.

Let $\Omega \subset R^n$ is a bounded domain with the sufficient smooth boundary $\Gamma = \partial\Omega,\ T > 0,\ Q = \Omega \times (0,T)$

Is a cylinder with the base, Ω, $\sum = \Gamma \times (0,T)$ is side of the cylinder Q.

Consider the following initial – boundary problem with the free boundary: for given $f \in L_2(0,T;L_2(\Omega))$ and $u_0 \in L_2(\Omega),\ u_0|_\Gamma \geq 0$ find the function

$u \in W_2^1\left(0,T;W_2^2(\Omega)\right)$, for which has a place the system

$$L\frac{\partial u}{\partial t} + Mu = f(x,t),\ (x,t) \in Q \tag{14}$$

$$u|_{t=0} = u_0(x) \tag{15}$$

$$u|_{\sum} \geq 0 \tag{16}$$

$$\left.\left(\frac{\partial}{\partial \nu_L}\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \nu_M}\right)\right|_{\sum} \geq 0 \tag{17}$$

$$u \cdot \left.\left(\frac{\partial}{\partial \nu_L}\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \nu_M}\right)\right|_{\sum} = 0 \tag{18}$$

Where the operators L and M act from the space $L_2\left(0,T;W_2^2(\Omega)\right)$ to the space

$L_2(0,T;L_2(\Omega))$ by formulas

$$Lu = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right) + a_0(x)u, \tag{19}$$

$$Mu = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\big(b_i(x,\nabla u)\big) + b_0(x)u, \tag{20}$$

and the normal derivatives with respect to the operators L and M are defined as

$$\frac{\partial u}{\partial v_L} = \sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial}{\partial x_j}\cos(v, x_i), \tag{21}$$

$$\frac{\partial u}{\partial v_M} = \sum_{i=1}^{n} b_i(x,\nabla u)\cos(v, x_i). \tag{22}$$

The functions $a_{ij}(x),\ a_0(x),\ b_j(x,\xi),\ b_0(x)$ are continuous on

$\bar{\Omega},\ a_{ij}(x) = a_{ji}(x)$,

1. $\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \alpha_1|\xi|^2,\ \forall x\in\Omega,\ \forall \xi\in R^n, \alpha_1>0,$
2. $a_0(x) \geq \alpha_2, \forall x\in\Omega, \alpha_2>0,$
3. $\sum_{i=1}^{n}\big(b_i(x,\xi)-b_i(x,\eta)\big)(\xi_i-\eta_i) \geq \beta_1|\xi-\eta|^2,\ \forall x\in\Omega,\ \forall \zeta,$ $\eta\in R^n,\ \beta_1>0,$
4. $b_0(x) \geq \beta_2,\ \forall x\in\Omega,\ \beta_2>0,$
5. $\big|b_i(x,\zeta)-b_i(x,\eta) \leq \beta_3||\zeta-\eta\big|,\ \forall x\in\Omega,\ \forall \zeta,\eta\in R^n,\ i=1,\ldots,n,\ \beta_3>0.$

Denote by $K = \left\{u\in L_2\left(0,T;W_2^1(\Omega)\right),\ u|_{\sum} \geq 0\right\}.$

Introduce the space $U = L_2\left(0, T, W_2^1(\Omega)\right)$, dense set

$D = \left\{ u \in U \mid \frac{\partial u}{\partial t} \in U,\ u(x,0) = u_0(x) \right\}$ in it and the operators $\widehat{L},\ \widehat{M}$ acting from U to the adjoint space $U' = L_2\left(0, T, W_2^{-1}(\Omega)\right)$ by the formulas

$$\left(\widehat{L}u, v\right) = \int_0^T \int_\Omega \sum_{i,j=1}^n a_{ij}(x) \frac{\partial u}{\partial x_j} \frac{\partial v}{\partial x_i} dxdt + \int_0^T \int_\Omega a_0(x) uvdxdt,$$

$$\left(\widehat{M}u, v\right) = \int_0^T \int_\Omega \sum_{i=1}^n b_i(x, \nabla u) \frac{\partial v}{\partial x_i} dxdt + \int_0^T \int_\Omega b_0(x) uvdxdt,$$

Weaken the conditions putting on the solution u of the inequality (0.21) and using the new operators $\widehat{L}$ and $\widehat{M}$ define the variational inequalities problem corresponding to the initial-boundary value problem (0.21)-(0.25): for the given $f \in U'$ we have to find the function $u \in U$ such that

$$\begin{cases} \left(\widehat{L}\frac{\partial u}{\partial t}, v - u\right) + \left(\widehat{M}u, v - u\right) \geq (f, v - u), \forall v \in K \\ u \in K \cap D \end{cases} \qquad (23)$$

Theorem 4. The solution of the initial-boundary value problem (14)-(18) is of the variational inequality with $f \in L_2(0, T; L_2(\Omega))$ is sufficiently smooth $\left(u \in L_2\left(0, T; W_2^2(\Omega)\right)\right)$ then it will be the solution of the initial-boundary value problem.

In chapter 2 paragraph 2 the following theorem is begin proved.

Theorem 5. The strong solution of the variational inequality (the solution of the problem (23) is also a weak solution (is the solution of the problem (14)-(18)

In chapter 2 paragraph 3 existence, uniqueness and smoothness of the weak solution is being studied.

Theorem 6. Under the conditions (1 – 5) the variational inequality has unique solution (23) $f \in U'$ belong to the set D, then it is also the strong solution.

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Ամփոփում
Միավաշ Ղարբանիա Գազաֆերուդի

Որոշ սկզբնական –եզրային և վարիացիոն խնդիրներ պսևդոէվոլուցիոն հավասարումների համար

Ատենախոսությունը նվիրված է երկրորդ կարգի պսևդոէվոլուցիոն հավասարումների համար որոշ սկզբնական-եզրային և վարիացիոն խնդիրների ուսումնասիրությանը: Ուսումնասիրվել են ինչպես չվերասերվող, այնպես էլ վերասերվող պսևդոէվոլուցիոն հավասարումներ:

Ներկայացվող ատենախոսության մեջ ստացվել են հետևյալ հիմնական արդյունքները:

- Ապացուցվել է, որ հետևյալ սկզբնական-եզրային խնդիրը

$$L\left(\frac{\partial^2 u}{\partial t^2}\right)+M(u)=0, \qquad (1)$$

$$u|_{t=0}=u^{(0)}(x), \quad u_t|_{t=0}=u^{(1)}(x), \qquad (2)$$

$$u|_{\Gamma^*}=0 \quad t>0 \qquad (3)$$

որտեղ

$$L(u)=-\sum_{i,j=1}^{n-1}\frac{\partial}{\partial x_i}\left(b_{ij}(x,t)\frac{\partial u}{\partial x_j}\right)-\frac{\partial}{\partial x_n}\left(b_{nn}(x)\frac{\partial u}{\partial x_n}\right),$$

$$M(u)=-\sum_{i,j=1}^{n-1}\frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_j}\right)-\frac{\partial}{\partial x_n}\left(a_{nn}(x,t)\frac{\partial u}{\partial x_n}\right).$$

Γ^*-ը տիրույթի եզրագծի մասն է (կարող է նաև համընկնել եզրագծի հետ կախված $b_{nn}(x,t)$ -ի վերասերման կարգից) համապատասխան ֆունկցիոնալ տարածությունում ունի միակ լուծում:

- Օգտագործելով Գալյորկինի մեթոդը, ապացուցվել է, որ պսևդոպարաբոլական հավասարման համար հետևյալ խնդիրը

$$\frac{\partial}{\partial t}L(u(t,x))+M(u(t,x))=f(t,x), \quad t>0, \; x=(x_1,\ldots,x_n)\in\Omega\subset\Re^n$$

$$u|_{\partial\Omega}=0$$

$$(Lu)(0,x)=g(x), \quad x\in\Omega,$$

որտեղ L -ը և M-ը 2-րդ կարգի չվերասերվող դիֆերենցիալ օպերատորներ են, ունի համապատասխան ֆունկցիոնալ տարածությունում լուծում՝ այն էլ միակը:

- Ուսումնասիրվել է հետևյալ վարիացիոն խնդիրը պսևդոպարաբոլական հավասարումների համար

$$L\frac{\partial u}{\partial t}+Mu=f(x,t),\ (x,t)\in Q$$

$$u|_{t=0}=u_0(x)$$

$$u|_{\sum}\geq 0$$

$$\left.\left(\frac{\partial}{\partial\nu_L}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial\nu_M}\right)\right|_{\sum}\geq 0$$

$$u\cdot\left.\left(\frac{\partial}{\partial\nu_L}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial\nu_M}\right)\right|_{\sum}=0$$

որտեղ

$$Lu=-\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right)+a_0(x)u,$$

$$Mu=-\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(b_i(x,\nabla u)\right)+b_0(x)u,$$

Ապացուցվել է այդ խնդրի համար թույլ լուծման գոյությունը, իսկ որոշ լրացուցիչ պայմանների դեպքում ուժեղ լուծման գոյությունը և միակությունը:

Заключение

Некоторые начально-краевые и вариационные задачи для псевдоэволюционных уравнений

Диссертационная работа посвящена исследованию некоторых начально-краевых вариационных задач для псевдоэволюционных уравнений. Изучены как невырожденные, так и вырожденные псевдоэволюционные уравнения.

В представленной работе получены следующуе основные результаты.

- Доказано, что следующая начально-краевая задача

$$L\left(\frac{\partial^2 u}{\partial t^2}\right)+M(u)=0, \tag{1}$$

$$u|_{t=0}=u^{(0)}(x), \quad u_t|_{t=0}=u^{(1)}(x), \tag{2}$$

$$u|_{\Gamma^*}=0 \quad t>0 \tag{3}$$

где

$$L(u)=-\sum_{i,j=1}^{n-1}\frac{\partial}{\partial x_i}\left(b_{ij}(x,t)\frac{\partial u}{\partial x_j}\right)-\frac{\partial}{\partial x_n}\left(b_{nn}(x)\frac{\partial u}{\partial x_n}\right),$$

$$M(u)=-\sum_{i,j=1}^{n-1}\frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_j}\right)-\frac{\partial}{\partial x_n}\left(a_{nn}(x,t)\frac{\partial u}{\partial x_n}\right).$$

имеет единственное решение в соответствующем функциональном пространстве, а Г *- в зависимости от вырождения функции $b_{nn}(x,t)$ совпадает со всей границей области или ее частью.

- Используя метод Галеркина, доказано, что для псевдопараболического уравнения следующая задача в соответствующем функциональном пространстве имеет единственное решение, где L и M невырожденные дифференциальные операторы.

$$\frac{\partial}{\partial t}L(u(t,x))+M(u(t,x))=f(t,x), \quad t>0,\ x=(x_1,\ldots,x_n)\in\Omega\subset\Re^n$$

$$u|_{\partial\Omega}=0$$

$$(Lu)(0,x)=g(x), \quad x\in\Omega,$$

- Исследована также следующая вариационная задача для псевдопараболического уравнения.

$$L\frac{\partial u}{\partial t}+Mu=f(x,t),\ (x,t)\in Q$$

$$u|_{t=0}=u_0(x)$$

$$u|_{\Sigma}\geq 0$$

$$\left.\left(\frac{\partial}{\partial \nu_L}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial \nu_M}\right)\right|_{\Sigma}\geq 0$$

$$u\cdot\left.\left(\frac{\partial}{\partial \nu_L}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial \nu_M}\right)\right|_{\Sigma}=0$$

где

$$Lu=-\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right)+a_0(x)u,$$

$$Mu=-\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(b_i(x,\nabla u)\right)+b_0(x)u,$$

Доказано, что эта задача имеет слабое решение, а при некоторых дополнительных условиях также доказано существование и единственность сильного решения.