ՀՀ ԳԱԱ ՄԵԽԱՆԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ

ԴԱՐՅԱԴԱՐ ՄՈՀԱՄՄԱԴ ԽԱԼԵՂՎԵՐԴԻ

ԲԱԶՄԱՇԵՐՏ ՍԱԼԵՐԻ ԵՎ ԹԱՂԱՆԹՆԵՐԻ ԿԱՅՈՒՆՈՒԹՅԱՆ, ՏԱՏԱՆՈՒՄՆԵՐԻ ԵՎ ՕՊՏԻՄԱԼ ՆԱԽԱԳԾՄԱՆ ԽՆԴԻՐՆԵՐ

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ՍԵՂՄԱԳԻՐ

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INSTITUTE OF MECHANICS OF NAS RA

DARYADAR MOHAMMAD KHALEGHVERDI

PROBLEMS OF STABILITY, VIBRATIONS AND OPTIMAL DESIGN OF MULTYLAYERED PLATES AND SHELLS

SYNOPSIS

of thesis for the request of the scientific degree of candidate of technical sciences by speciality 01.02.04-“Mechanics of deformable solid body”

Yerevan– 2019

Ատենախոսության թեման հաստատվել է Երևանի Պետական Համալսարանում

Գիտական ղեկավար — տեխ. գիտ. դոկտոր, պրոֆ. Լ.Ա. Մովսիսյան
Պաշտոնական ընդդիմախոսներ — տեխ. գիտ. դոկտոր, պրոֆ. Է.Վ. Բելուբեկյան
տեխ. գիտ. դոկտոր Ս.Պ. Ստեփանյան

Առաջատար կազմակերպություն — Հայաստանի ազգային պոլիտեխնիկական համալսարան

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Ատենախոսությանը կարելի է ծանոթանալ ՀՀ ԳԱԱ Մեխանիկայի ինստիտուտի գրադարանում

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The defence of the thesis will take place on September 6, 2019, at 14.00 at the meeting of the specialized council of Mechanics – 047/19-01 at the Institute of Mechanics

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The thesis can be get aquainted with at the library of the Institute of Mechanics NAS RA

The abstract is sent July,11. 2019թ.

Scientific secretary of the
Specialized council d.ph.math.sci. — A.V.Sahakyan

GENERAL DESCRIPTION OF WORK

The Actuality of the thesis. Multylayered plates and shells are parts of the modern engineering constructions, machines, aeroplanes, rockets: They are often made of anisotropic composite materialsThe study oft he questions of stability, vibrations and optimal design oft hin-walled constructions represents a great interest and is an object of studies nowdays.In case of application of composite materials the problem, in which the anisotropic and inhomogeneous properties of the constructions are taken into account, are of great importance. The questions, related to the existence and equivalence of the problems solutions and the development of the solutions new methods, are one of the important themes of the optimal project theory.

The thesis is devoted to one of the modern themes of Mechanica of deformable solid body - to the thin-walled constructions stability, vibrations and optimal design problems which have both theoretical and applied significance. In the paper on the base of the main provisions oft he optimal design modern theory a lot of problems of applied nature are solved.

In these problems the values characterizing the mass, rigidness, or stability are the perfection criteria of the shells, plates.

The goal of the thesis. To research:

- The problem of a rectangular plate stability in case of arbitrarily distributed forces, the problem of multilayered composite viscoelastic plate stability, the problems of rectangular plate and layered plate stability, taking into account transverse shears.
- Problems of shells optimal design, problems of diminishing, as much as possible, three-layered, one-layered shells masses, when a restriction is imposed on the basic frequency value of the shells free vibrations, bars optimization problem.
- Rectangular layered plate free vibrations problem, when all the sides of the plate are jointly fastened, plate vibrations and stability problems, taking into account the transverse shear.
- Problems of stability, free vibrations and optimal management of layered composite plates, made on the base of fiber filler material.

The scientific novelty of the work.

- One-layered rectangular plate stability problem in case of arbitrarilily distributed forces, composite viscoelastic multilayered rectangular plate stability problem, in case of the different arrangement of the layers towards the coordinate plane, when on both sides of the plate equally pressing forces are acting and two problems of the layered plate stability, taking into account the transverse shears.
- In cases of external axisymmetric arbitrary normal load, freely supported boundary and floating fastening boundary conditions, the problem of the cylinder shell stability is solved and the stability critical parameters are determined.
- Problems of a homogeneous anisotropic one-layered cylindrical shell with changing thickness, one-layered homogeneous orthotropic rotation shell with changing thickness and the optimal design of the open cylindrical shell, are solved. Problems of

bending, forced vibrations, stability and optimal management of plates, made of multilayered composite material, are solved.

The practical significance. The rectangular plates and shells are widely applied in modern engineering constructions and in other fields of technology. In the thesis the obtained results are up-to-date and can be applied in the new peoblems of multilayered rectangles and plates, as well as for design and calculation of engineering constructions, bridges, hydraulic constructions, aeroplanes, rockets, ships, machines parts, etc.

Approbation of the thesis results. The thesis main results have been reported and discussed:

- Modern problems of Mechanics of solid invironment in the IV international conference. Tsakhkadzor, Armenia (September, 21-26, 2015):
- In the I and II international conferences Iran-Armenia. Yerevan, Armenia (September, 16-17, 2011, June, 24-25, 2013):
- In the seminar “Engineering Mechanics” of the faculty of Mechanics, Tehran University (March, 2014):
- In the Mechanics department seminar of the faculty of Mathematics and Mechanics, Yerevan State University, Yerevan, Armenia (March,2011):
- In “Thin-walled systems Mechanics” department session and scientific seminar, Inst. Of Mechanics, NAS RA (December, 2018).
- In the seminar of the Inst. Of Mechanics, NAS RA (March, 2019).

Publications of the Author. On the theme of the thesis ten scientific papers have been published. The reference is attached to the end of the summary.

The structure and the volume of the work. The paper consists of the introduction, four chapters, conclusion and reference. The thesis contains 136 pages, 45 pictures, and 1 table.

Brief Content of the Work

In the introduction the thesis modernity and scientific novelty is justified, the main stages of stability, vibration and optimal design theory development is described, the role and significance of that theory is clarified.

The first chapter is devoted to one of the most significant problems of Mechanics of a deformable solid body- stability, vibration and optimal design problems of constructions. The main definitions and affiliations are brought and in case of the arbitrarily distributed forces, problems of rec tangular plate stability, multilayered composite viscoelstic plate stability, rectangular and layered plate stability, taking into account the transverse shears are studied. The optimal design problem of a three-layered anisotropic inhomogeneous plate is studied.

In the first paragraph the rectangular plate stability problem in case of arbitrary normally distributed forces with different boundary conditions is solved. Using the plate

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+p(y)\frac{\partial^2 w}{\partial x^2}=0 \quad (1)$$

bended surface equation, where` $D_{ij} = B_{ij}\frac{h^3}{12}$, are the rigidities of the anisotropic plate and from the following boundary conditions

a) Hinge fastening

$$w = 0, \quad M_x = \frac{\partial^2 w}{\partial x^2} = 0, \quad \text{when } x = 0 \text{ and } x = a \tag{2}$$

$$w = 0, \quad M_y = \frac{\partial^2 w}{\partial y^2} = 0, \quad \text{when } y = 0 \text{ and } y = b$$

b) two edges are hinge and on the other two ones the following boundary conditions are given

$$w = 0, \quad M_x = \frac{\partial^2 w}{\partial x^2} = 0, \quad \text{when } x = 0 \text{ and } x = a \tag{3}$$

$$\frac{\partial w}{\partial y} = \frac{\partial^3 w}{\partial y^3} = 0, \text{ when } y = 0 \text{ and } y = b,$$

The critical values of the forces are determined:

In case of the hinge boundary conditions w searchable bending is searched in the form of

$$w = sin\lambda_m x \sum_{n=1}^{\infty} f_n \, sin\mu_n y, \tag{4}$$

where $\lambda_m = \frac{m\pi}{a}$, $\mu_n = \frac{n\pi}{b}$, and $p(y)$ function is represented in the form of series

$$p(y) = \sum_{k=0}^{\infty} a_k \, cos\mu_k y, \; \mu_k = \frac{k\pi}{b} \tag{5}$$

Putting (4) and (5) into equation (1) we get an infinite equations system, the determinant of which being zero determines the critical value of $p(y)$ force.

In case of (3) boundary condition w bend is searched in the following form

$$w = sin\lambda_m x \sum_{n=0}^{\infty} f_n \, cos\mu_n y \tag{6}$$

In the edges forces distribution two private cases, when the external forces change by parabol law, is observed.

In the second paragraph the problem of a rectangular plate stability, made of viscoelastic materials, when equally pressing by two sides forces p act on it. In this case the stability equation has got

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} - D_1\Gamma^*\Delta^2 w - 2D_2\Gamma^*\frac{\partial^4 w}{\partial x^2 \partial y^2} + P\frac{\partial^2 w}{\partial x^2} = 0 \tag{7}$$

the form, where` $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

For viscoelastic materials stress-strain contact is expressed in the form of the following formula

$$\sigma_x = A_n\varepsilon_x + A_{12}\varepsilon_y, \; \sigma_y = A_{22}\varepsilon_x + A_{22}\varepsilon_y, \; \sigma_{xy} = \tilde{A}_{66}\varepsilon_{xy} \tag{8}$$

$$\tilde{A}_{66} f = A_{66}\left(A - \Gamma^*\right) f, \; \Gamma^* = D\int_0^t e^{-\alpha(t-\tau)} f(\tau)\, d\tau, \; D = \left(1 - \frac{A_{66}^{\infty}}{A_{66}}\right)\alpha \tag{9}$$

The problem for a plate supported freely is solved, the formulae for the calculation of instant and prolonged critical forces are obtained. The instant criticalforce is equal to

$$P_{кр}^{м} = A(D_{11} + D_{12} + 2D_{66}), A = \frac{2\pi^2}{a^2} \tag{10}$$

And the prolonged critical force

$$P_{кр}^{(д)} = P_{кр}^{(м)} - \frac{A}{\alpha}(2D_1 + D_2)D \tag{11}$$

In the third paragraph the problem of a rectangular plate stability, taking into account the transverse shear is observed. The following simple model is admitted

$$\boldsymbol{u_x} = u + z\varphi,\ \ \boldsymbol{u_y} = v + z\psi,\ \ \boldsymbol{u_z} = w \tag{12}$$

Two problems are observed.

The first one is the problem of a plate stability, when on two opposite sides equally distributed p forces act, and the second one is the problem of a layered plate stability, when on its external planes oppositely directed tangential stresses of S intensity act.

In the first problem for a uniformal (cylindrical bend) problem a simple solution is obtained

$$P_{kp} = D_{11}{\lambda_1}^2\left(1 - \frac{D_{11}}{C_{44}}{\lambda_1}^2\right) \tag{13}$$

From here it is seen that the calculation of the transverse shears brings to the minimization of the critical force and when $C_{44} \to \infty$ we get $P_{kp} = D_{11}{\lambda_1}^2$ result by classical theory.

In the second problem, the preliminary of which is momentary, the critical force is determined with the following formula

$$Sh = \pm\sqrt{C_{11}D_{11}}\,\lambda_1 \tag{14}$$

In fact, in a uniformal problem the account of the tranverse shears does not bring to the value change, obtained by classical theory.

In the fourth paragraph an optimal design problem is observed: to find such three-layered plate $h_ш$ thickness, in case of which the plate mass gets the smallest value, and the value of free vibrations basic frequency is ω_0: It is admitted that the edges of the plate are hingely fastened.

The equation, describing the free plate vibrations, is

$$\frac{\partial^2}{\partial x^2}\left(D_{11}\frac{\partial^2 w}{\partial x^2} + D_{12}\frac{\partial^2 w}{\partial y^2} + D_{16}\frac{\partial^2 w}{\partial x \partial y}\right) + 2\frac{\partial}{\partial x \partial y}\left(D_{16}\frac{\partial^2 w}{\partial x^2} + D_{26}\frac{\partial^2 w}{\partial y^2} + D_{66}\frac{\partial^2 w}{\partial x \partial y}\right) +$$
$$+\frac{\partial^2}{\partial y^2}\left(D_{12}\frac{\partial^2 w}{\partial x^2} + D_{22}\frac{\partial^2 w}{\partial y^2} + D_{26}\frac{\partial^2 w}{\partial x \partial y}\right) = 2(\rho_0 h_0 + \rho_ш h_ш)\frac{\partial^2 w}{\partial t^2} \tag{15}$$

The problem to find the smallest value of the plate mass is brought

$$J = \int_0^a \int_0^b h_ш(x,y)\,dx\,dy \tag{16}$$

To the problem of finding the functional smallest value. We should find $h_ш(x,y)$ such a value which satisfies equation (15), (2) boundary conditions and incase of $\omega = \omega_0$ informs the smallest value to functional (16). The problem is solved by variation method. It is brought to non-linear differential equations system with partial derivatives

W, $h_{ш}$ towards the unknown functions. The system is solved by numerical method – using Matlab program. The results are reflected in the form of graphs.

The second chapter is devoted to the problems of the shells optimal design. Problems of three-layered and one-layered shells masses as much as possible minimization, when a restriction is set on the basic frequency value of the shells free vibrations. The optimization problem of bars is observed, too. As an example the problem of as much as possible minimization of a circular bar biggest bending under a concentrated force action is observed. The solution of the anisotropic cylindrical shells optimal design problem is brought to non-linear differential equations system, which is solved by a numerical method, using Mutlub program.

In the first paragraph the problem of cylindrical shells stability in case of external axisymmetric arbitrarily normal load is studied. The equation system, corresponding to the preliminary state, will be

$$C_{11}\frac{d^2u_0}{dx^2}+C_{12}\frac{1}{R}\frac{dw_0}{dx}=0 \tag{17}$$

$$D_{11}\frac{d^4w_0}{dx^4}+\frac{1}{R}(C_{12}\frac{du_0}{dx}+C_{22}\frac{w_0}{R})=Z(x)$$

Stability state equations

$$C_{11}\frac{\partial^2u}{\partial x^2}+C_{12}(\frac{\partial^2v}{\partial x\partial y}+\frac{1}{R}\frac{\partial w}{\partial x})+C_{66}(\frac{\partial^2u}{\partial y^2}+\frac{\partial^2v}{\partial x\partial y})=0$$

$$C_{65}(\frac{\partial^2u}{\partial x\partial y}+\frac{\partial^2v}{\partial x^2})+C_{12}\frac{\partial^2u}{\partial x\partial y}+C_{22}(\frac{\partial^2v}{\partial y^2}+\frac{1}{R}\frac{\partial w}{\partial y})=0 \tag{18}$$

$$D_{11}\frac{\partial^4w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4w}{\partial y^4}+\frac{1}{R}[C_{12}\frac{\partial u}{\partial x}+C_{22}(\frac{\partial v}{\partial y}+\frac{w}{R})]-\frac{\partial}{\partial x}(T_1{}^0\frac{\partial w}{\partial x})-\frac{\partial}{\partial y}(T_2{}^0\frac{\partial w}{\partial y})=0$$

Two case of boundary conditions have been discussed.

1) $w=\frac{\partial^2w}{\partial x^2}=T_1=v=0$ when $x=0$ and $x=l$ (19)

2) $\frac{\partial w}{\partial x}=\frac{\partial^3w}{\partial x^3}=u=s=0$ when $x=0$ and $x=l$ (20)

The first boundary conditions correspond to the classic free supported case, and the second one – to the case of "floating fastening"

The solution of the first boundary problem is searched in the form of

$$v=\sin\mu_ny\sum_{m=1}^{\infty}v_m\sin\lambda_mx \tag{21}$$

$$w=\cos\mu_ny\sum_{m=1}^{\infty}w_m\sin\lambda_mx\ ,\quad \lambda_m=\frac{m\pi}{l}\ ,\ \mu_n=\frac{n}{R}$$

And in case of the second boundary problem it has the form

$$u=\cos\mu_ny\sum_{m=1}^{\infty}u_m\sin\lambda_mx\ , \tag{22}$$

$$v=\sin\mu_ny\sum_{m=0}^{\infty}v_m\cos\lambda_mx,\ w=\cos\mu_ny\sum_{m=0}^{\infty}w_m\cos\lambda_mx$$

It is admitted that forces T_1^0 and T_2^0 have got the folloing form

$$T_1^0=\sum_{k=0}^{\infty}b_k\cos\lambda_nx\ ,\qquad T_2^0=\sum_{n=0}^{\infty}a_n\cos\lambda_nx \tag{23}$$

The critical parameters of stability are determined.

In the second paragraph the optimal design problem of homogeneous anisotropic one-layered cylindrical shell of changing thickness is observed and the purpose is to find such a thickness of the shell in order to have the smallest value for the plate mass. The free vibrations of such a shell are described by the system of classical differential equations.

It is assumed that in $\alpha = 0$, $\alpha = L$ borders one of the following conditions takes place:

a) free edge.

$$T_1 \equiv C_{11}\frac{dU}{d\alpha} + C_{12}\frac{W}{R} + C_{16}\frac{dV}{d\alpha} = 0\,, \quad S_{12} \equiv C_{16}\frac{dU}{d\alpha} + C_{26}\frac{W}{R} + C_{66}\frac{dV}{d\alpha} = 0, \tag{24}$$

$$M_1 \equiv -D_{11}\frac{d^2W}{d\alpha^2} + \frac{2D_{16}}{R}\frac{dV}{d\alpha} = 0, \ \ N_1 \equiv \frac{dM_1}{d\alpha} = 0$$

b) hinge fastened edge

$$U = 0, \quad V = 0, \quad W = 0, \quad M_1 = 0 \tag{25}$$

c) hinge fastened but free edge in the tangential direction

$$T_1 = 0, \quad V = 0, \quad W = 0, \quad M_1 = 0 \tag{26}$$

The shell mass with the exactness of a constant multiplier is given with the help of functional

$$J = \int_0^l h(\alpha)d\alpha \tag{27}$$

The problem is solved by the variation method. It is brought to a close system of non-linear equations $U(\alpha)$, $V(\alpha)$, $W(\alpha)$, $h(\alpha)$ towarda the unknown functions and this system is solved with the numerical method – with the help of Matlab program.

In the third paragraph a problem of minimizing as much as possible the biggest bent of a bar bending with circular cut under the action of a concentrated force. The circular bar of $2l$ length is hingely fastened at $x = \pm l$ edges and on it at $x = 0$ point the concentrated force $q = P \cdot \delta(x)$, where $\delta(x)$-is Dirac function, acts.

The following problem of a bar optimization having circular cross-section with $r(x)$ changing radius is observed: to find such $h(x) = r^2(x)$ function describing the changing cut of the circular bar cross-section, in the case of which the biggest bent of the bar has got the smallest possible value, and the bar volume accepts a predetermined value ($r(x)$-is the radius of the cross-section).

With the help of $h(x)$ function $S(x)$ surface and $D = EI(x)$ rigidity of the circular bar cross-section is determined

$$S(x) = \pi r^2 = \pi h(x) = B_2 h(x), \ \ EI(x) = \frac{E\pi}{4}r^4 = \frac{E\pi}{4}h^2(x) = A_2 h^2(x) \tag{28}$$

where $B_2 = \pi, A_2 = \frac{E\pi}{4}, I(x)$-is the moment of the section inertia, E-is Young module of the bar material.

The basic concerns of the formulated problem are written in the following form:

$$(DW_{xx})_{xx} = q, \quad W\big|_{x=\pm l} = DW_{xx}\big|_{x=\pm l} = 0\,, \int_{-l}^{l} S(x)dx = V_0 \tag{29}$$

$$J = \int_{-l}^{l} W(x)\delta(x)dx = W(0) \xrightarrow[h(x)]{} \min,$$

where $W(x)$-is the function of the bar bent, J-is the goal fuctional.

In the formulated problem it is required to find the bar cross-section changing cut characterizing such a law, that the biggest bent of the bar could get the smallest value, and the volume could accept the beforehand given value. Formulae of the bar bent and optimal thickness is obtained.

In the fourth paragraph the optimal design problem of one-layered h with changing thickness and with r radius of homogeneous orthotropic rotation shell is observed – to find such thickness of the shell, in case of which the shell mass gets the smallest value.

On $S = S_0$ and $S = S_1$ borders of the shell the following conditions take place.

$$W(S) = \frac{dW}{dS} = U(S) = 0, \tag{30}$$

where $U(\alpha)$, $W(\alpha)$-are the displacements. The shell mass with the constant factor exactness is equivalent to the following integral.

$$J = \int_{S_0}^{S} rhdS \tag{31}$$

The problem is solved with a variation method. Non-linear differential equations system, which is solved with numerical method – Matlab program, and the results are illustrated in the form of grapghs, is obtained.

In the fifth paragraph the problem of an open cylindrical shell optimal design is observed: to find such shell $h(\alpha,\beta)$ thichness, in case of which with free vibrations basic ω_0 fixed frequency value the shell mass obtains the smallest value: $\alpha = 0$, $\alpha = a$, $\beta = 0$, $\beta = b$ in the borders one of the following boundary conditions is given.

a) free edge

$$T_1 = 0, \quad S_{12} + \frac{H_{12}}{R} = 0, \quad N_1 + \frac{\partial H_{12}}{\partial \beta} = 0, \quad M_1 = 0 \tag{32}$$

b) hingely fastened edge

$$U = 0, \quad V = 0, \quad W = 0, \quad M_1 = 0 \tag{33}$$

c) hinge, in tangential direction free edge

$$V = 0, \quad W = 0, \quad T_1 = 0, \quad M_1 = 0 \tag{34}$$

d) hinge in normal direction free edge

$$M_1 = 0, \quad N_1 + \frac{\partial H_{12}}{\partial \beta} = 0, \quad U = 0, \quad V = 0 \tag{35}$$

e) rigidly fastened edge

$$U = 0, V = 0, W = 0, \tilde{\theta} = -\frac{\partial W}{\partial \alpha} = 0 \tag{36}$$

The shell mass with the multiplier exactness is given with the help of the following functional:

$$J = \int_0^a \int_0^b h(\alpha,\beta)\, d\beta\, d\alpha \tag{37}$$

The problem is solved by the variation method. As in previous problems, the non-linear equation system, which is solved by the numerical method, is obtained.

In the third chapter the problems of the shells free vibrations and stability are studied. The problem of rectangular layered plate free vibrations, when all the sides are hingely fastened. Problems of a layered orthotropic rectangular plate stability, when all the sides of the plate are hingely fastened, and when two opposite sides are hingely fastened, and on the other two sides arbitrary boundary conditions are given, are studied. The problem of plate vibrations and stability, taking into account of the transverse shears, is observed. The problems of plate dynamic stability and bent problems are solved, too.

In the first paragraph the problem of free vibrations of a rectangular layered plate with $a \times b$ sizes, when all the sides of the plate are hingely fastened. The equation of a layered plate free vibrations is:

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} + 2N\rho h\frac{\partial^2 w}{\partial t^2} = 0 \tag{38}$$

Here ρ-is the density of the material, h-is the thickness of one layer, N–is the number of the layers.

The case, when all the sides of the rectangular plate are hingely fastened, the boundary conditions are given by formulae (2), is studied. w bent should satisfy the following initial conditions, too

$$w = w_0(x,t)\ ,\ \ \frac{\partial w}{\partial t} = v_0(x,t),$$

where w_0, v_0-are the initial bent and velocity.

Bent w, satisfying conditions (2) is searched in the following form

$$w = f_{mn}e^{i\omega t} sin\lambda_m x\, sin\mu_n y,\quad \lambda_m = \frac{m\pi}{a},\quad \mu_n = \frac{n\pi}{b} \tag{39}$$

For free vibrations frequencies we have

$$\omega_{mn} = \frac{\pi^2}{b^2}\sqrt{\frac{1}{2n\rho h}}\ \sqrt{D_{11}\left(\frac{m}{c}\right)^4 + 2(D_{12} + 2D_{66})n^2\left(\frac{m}{c}\right)^2 + D_{22}n^4}\ ,\ \ c = \frac{a}{b} \tag{40}$$

In a private case, for a square $(a = b)$ plate, when $c = 1$,

$$\omega_{mn} = \frac{\pi^2}{a^2}\sqrt{\frac{1}{2n\rho h}}\ \sqrt{D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2 + D_{22}n^4} \tag{41}$$

And the first smallest frequency will be

$$\omega_{11} = \frac{\pi^2}{a^2}\sqrt{\frac{1}{2n\rho h}}\ \sqrt{D_{11} + 2(D_{12} + 2D_{66}) + D_{22}} \tag{42}$$

In the second paragraph, taking into account the transverse shears, two problems of a layered plate stability are observed.

I. We have got with four sides hingely fastened rectangular plate, which is subjected to compression P by unformly distributed normal forces. The critical force is determined by the equation

$$D_1\frac{\partial^4 w}{\partial x^4}+2D_3\frac{\partial^4 w}{\partial x^2\partial y^2}+D_2\frac{\partial^4 w}{\partial y^4}+P\frac{\partial^2 w}{\partial x^2}=0 \tag{43}$$

When four sides of the plate are hingely fastened, the solution of equation (43) is searched in the form of

$$w=f_{mn}sin\lambda_m x\, sin\mu_n y,\quad \lambda_m=\frac{m\pi}{a},\quad \mu_n=\frac{n\pi}{b} \tag{44}$$

(43) is not the exact zero solution of boundary problem (2)

$$P\left(\frac{m}{a}\right)^2=\pi^2\left[D_1\left(\frac{m}{a}\right)^4+2D_3\left(\frac{mn}{ab}\right)^2+D_2\left(\frac{n}{b}\right)^4\right] \tag{45}$$

P- value, determined by formula (45), according to n will be minimum, when $n=1$

$$P=\frac{\pi^2\sqrt{D_1D_2}}{b^2}\left[\sqrt{\frac{D_1}{D_2}}\left(\frac{m}{c}\right)^2+\frac{2D_3}{\sqrt{D_1D_2}}+\sqrt{\frac{D_2}{D_1}}\left(\frac{c}{m}\right)^2\right]\text{ where` }\quad c=\frac{a}{b}$$

II. When the opposite sides of the plate are hingely fastened, and on the other two sides arbitrary conditions are given, then the solution of equation (42) is searched in the form of

$$w=f(y)\,sin\lambda_m x,\quad \left(\lambda_m=\frac{m\pi}{a}\right) \tag{46}$$

where $f(y)$-is an unknown function. For the determination of the unknown function $f(y)$ we get

$$D_2f^{IV}-2\lambda_m^2D_3f''+(D_1\lambda_m^2-P)\lambda_m^2 f=0 \tag{47}$$

The differential equation, the characteristic equation roots of which are $\pm ik_1, \pm ik_2$, where

$$k_1=\sqrt{\lambda_m\sqrt{\left(\frac{D_3}{D_2}\right)^2\lambda_m^2+\frac{P}{D_2}+\frac{D_3-D_1}{D_2}\lambda_m^2}},\ k_2=\sqrt{\lambda_m\sqrt{\left(\frac{D_3}{D_2}\right)^2\lambda_m^2-\frac{D_1+D_3}{D_2}\lambda_m^2+\frac{P}{D_2}}} \tag{48}$$

Therefore, the solution of equation (47) will have the following form

$$f(y)=Achk_1y+Bshk_1y+C\,cosk_2y+Dsink_2y \tag{49}$$

Satisfying the boundary conditions given on $y=0$ and $y=b$ sides for A,B,C and D constants we obtain homogeneous equations system, by solvable condition of which the critical force volume P is determined.

In a private case, when all the sides are fastened

$$w=\frac{\partial w}{\partial x}=0,\quad \text{wnen } x=0 \text{ and } x=a$$

$$w=\frac{\partial w}{\partial y}=0,\quad \text{wnen } y=0 \text{ and } y=b \tag{50}$$

In order to find P critical force a transcentend equation is obtained

$$thk_1b\ thk_2b = \frac{2k_1k_2}{k_1^2-k_2^2}\left(1-\frac{1}{chk_1b\ cosk_2b}\right) \quad (51)$$

III. When four sides of the plate are under the action of an external load, particularly, on $x = 0$ and $x = a$ edges p_1 uniformly distributed forces act, and on $y = 0$ and $y = b$ edges p_2 uniformly distributed forces, then the perturbed state bent should satisfy the following equation

$$D_1\frac{\partial^4 w}{\partial x^4} + 2D_3\frac{\partial^4 w}{\partial x^2\partial y^2} + D_2\frac{\partial^4 w}{\partial y^4} + p_1\frac{\partial^2 w}{\partial x^2} + p_2\frac{\partial^2 w}{\partial y^2} = 0 \quad (52)$$

From hinge boundary conditions we shall have

$$p_1\left(\frac{m}{a}\right)^2 + p_2\left(\frac{n}{b}\right)^2 = \pi^2\left[D_1\left(\frac{m}{a}\right)^4 + 2D_3\left(\frac{mn}{ab}\right)^2 + D_2\left(\frac{n}{b}\right)^2\right] \quad (53)$$

For the certainty of the problem setting additional conditions p_1 and p_2 for the forces should be given. Some private cases are observed.

1. p_1 and p_2 forces are different, but relationship is constant

$$p_1 = \lambda,\ p_2 = \lambda\alpha \quad (54)$$

In that case λ the critical value is equal to

$$\lambda = \frac{\pi^2}{b^2}\frac{D_1\left(\frac{m}{c}\right)^2 + 2D_3n^2 + D_2\left(\frac{c}{m}\right)^2 n^4}{1-\alpha\left(\frac{c}{m}\right)^2 n^2} \quad (55)$$

2. If in $x = 0$ and $x = a$ edges uniformly distributed compressing λ forces act, and on $y = 0$ and $y = b$ edges p -stretching forces, moreover, the volume of the compressing forces can change, but the volume of the stretching forces cannot change, the action of the stretching forces can be estimated. In that case from (53) we shall have

$$\lambda = \frac{\pi^2}{b^2}\left[D_1\left(\frac{m}{c}\right)^2 + 2D_3n^2 + D_2\left(\frac{c}{m}\right)^2 n^4 + \frac{\rho b^2}{\pi^2}\left(\frac{c}{m}\right)^2 n^2\right] \quad (56)$$

The smallest value of λ -will be obtained, when $n = 1$, and $c = \frac{m'}{\sqrt[4]{\frac{D_2}{D_1} + \frac{pb^2}{\pi^2 D_1}}}$ lı

$$\lambda_{kp} = \frac{2\pi^2}{b^2}\sqrt{D_1D_2}\left(\sqrt{1+\frac{pb^2}{\pi^2 D_2}} + \frac{D_3}{\sqrt{D_1D_2}}\right) \quad (57)$$

In the third paragraph the problem of plate vibration, taking into account the transverse shears, is observed. The tranverse shears and displacement components, according to the hight of the plate, change by the following laws

$$\tau_{xz} = f(z)\varphi(x,y,t),\ \tau_{yz} = f(z)\psi(x,y,t),\ u_z = w(x,y,t) \quad (58)$$

$$u_x = w(x,y,t) - z\frac{\partial w}{\partial x} + a_{55}\frac{z}{2}\left(\frac{h^2}{4}-\frac{z^2}{3}\right)\varphi,\ u_y = v(x,y,t) - z\frac{\partial w}{\partial y} + a_{44}\frac{z}{2}\left(\frac{h^2}{4}-\frac{z^2}{3}\right)\psi$$

Let's consider the plate free vibrations, which are represented by the following system

$$\frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y} = \frac{h^3}{12}\rho h\frac{\partial^2 w}{\partial t^2}$$

$$B_{11}\frac{\partial^3 w}{\partial x^3}+(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x\partial y^2}-\frac{h^2}{10}\left[a_{55}\left(B_{11}\frac{\partial^2\varphi}{\partial x^2}+B_{66}\frac{\partial^2\varphi}{\partial y^2}\right)+a_{44}(B_{12}+B_{66})\frac{\partial^2\psi}{\partial x\partial y}\right]+\varphi=0 \tag{59}$$

$$B_{22}\frac{\partial^3 w}{\partial y^3}+(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x^2\partial y}-\frac{h^2}{10}\left[a_{44}\left(B_{22}\frac{\partial^2\psi}{\partial y^2}+B_{66}\frac{\partial^2\psi}{\partial x^2}\right)+a_{55}(B_{12}+B_{66})\frac{\partial^2\varphi}{\partial x\partial y}\right]+\psi=0$$

Let's consider the case, when all four sides of the plate are hingely fastened

$w=M_x=\psi=0$, when $x=0$ and $x=a$

$$w=M_y=\varphi=0,\ \text{when}\ \ y=0\ \text{and}\ y=b \tag{60}$$

The solution of system (59) is searched in the following form

$$w=f_{mn}sin\lambda_m x\, sin\mu_n y\, cs\omega_{mn}t,\quad \varphi=\varphi_{mn}cos\lambda_m x\, sin\mu_n y\, cos\omega_m t \tag{61}$$

$$\psi=\psi_{mn}sin\lambda_m x\, cos\mu_n y\, cos\omega_m t,\quad \lambda_m=\frac{m\pi}{a},\quad \mu_n=\frac{n\pi}{b}$$

For the frequency we obtain

$$\omega_{mn}=\omega_{mn}^0\sqrt{1-d} \tag{62}$$

Here ω_{mn}^0 - is the frequency of the free vibrations obtained from classical theory

$$\omega_{mn}^0=\sqrt{\frac{1}{\rho h}}\,[D_{11}\lambda_m^4+2(D_{12}+2D_{66})\lambda_m^2\mu_n^2+D_{22}\mu_n^4]^{1/2},\quad d=\frac{B-A}{1+B} \tag{63}$$

As $0<d<1$, we obtain that, taking into account with shears, the frequency is small from the determined one with classical theory.

In the fourth paragraph the problem of the rectangular plate stability, taking into account the transverse shears, determined by the beforehand stress-strain state

$$T_1^0=-p,\ T_2^0=-\lambda p \tag{64}$$

For the determination of the plate critical load we use the second and third equation of system (60) and equation

$$\frac{h^3}{12}\left(\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial y}\right)-p\left(\frac{\partial^2 w}{\partial x^2}+\lambda\frac{\partial^2 w}{\partial y^2}\right)=0 \tag{65}$$

The solution of the system is searched in the form of (61), and the boundary conditions are given with formulae (60). The critical value of the load is determined

$$p_{mn}^*=p_{mn}^0\sqrt{1-d} \tag{66}$$

By the formula, where when $0<d<1$, and

$$p_{mn}^0=\frac{D_{11}\lambda_m^4+2(D_{12}+2D_{66})\lambda_m^2\mu_n^2+D_{22}\mu_n^4}{\lambda_m^2+\lambda\mu_n^2} \tag{67}$$

From formula (66) it is seen, that calculated by the precised theory the value of the critical force is smaller from the obtained value of the problem by classical setting. In the private case, when $\lambda=0$

$$p_{mn}^*=\frac{1}{\lambda_m^2}[D_{11}\lambda_m^4+2(D_{12}+2D_{66})\lambda_m^2\mu_n^2+D_{22}\mu_n^4]\frac{1+A}{1+B} \tag{68}$$

In the fifth paragraph the problem of the plate dynamic stability, when on its plane acting force is periodical function of time $T_1^0=-p(t)$.

The system studying the dynamic stability will be

$$\frac{h^2}{12}\left(\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial y}\right)-p(t)\frac{\partial^2 w}{\partial x^2}-\rho h\frac{\partial^2 w}{\partial t^2}=0 \tag{69}$$

$$\left[B_{11}\frac{\partial^3 w}{\partial x^3}+(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x\partial y^2}\right]-\frac{h^2}{10}\left[a_{55}\left(B_{11}\frac{\partial^2\varphi}{\partial x^2}+B_{66}\frac{\partial^2\varphi}{\partial y^2}\right)+a_{44}(B_{12}+B_{66})\frac{\partial^2\psi}{\partial x\partial y}\right]+\varphi=0$$

$$\left[B_{22}\frac{\partial^3 w}{\partial y^3}+(B_{12}+2B_{66})\frac{\partial^3 w}{\partial y\partial x^2}\right]-\frac{h^2}{10}\left[a_{44}\left(B_{22}\frac{\partial^2\psi}{\partial y^2}+B_{66}\frac{\partial^2\psi}{\partial x^2}\right)+a_{55}(B_{12}+B_{66})\frac{\partial^2\varphi}{\partial x\partial y}\right]+\psi=0$$

The boundary conditions are represented by formula (60). The solution is searched in the following form

$$w=f_{mn}(t)sin\lambda_m x\, sin\mu_n y$$

$$\varphi=\varphi_{mn}(t)cos\lambda_m x\, sin\mu_n y\,,\ \psi=\psi_{mn}(t)sin\lambda_m x\, cos\mu_n y \tag{70}$$

When $p(t)$- is a periodical function` $p(t)=p_0-p_1cos\theta t$, then for the determination of $f_{mn}(t)$ we get

$$\frac{d^2 f_{mn}(t)}{dt^2}+\Omega_{mn}^2(1-2\mu_{mn}cos\theta t)f_{mn}=0 \tag{71}$$

The differential equation, where

$$\Omega_{mn}=\omega_{mn}\left(\frac{1-p(t)}{p_{mn}^*}\right)^{1/2},\quad \mu_{mn}=\frac{pt}{2(p_{mn}^*-p_0)} \tag{72}$$

Equation (71) is a well-known Matier equation, for which stability and unstability areas are well-known.

When $p_0=0\,,\ -p(t)=p_1cos\theta t$, then equation (71) will have

$$\frac{d^2 f_{mn}(t)}{dt^2}+\omega_{mn}^2(1-2\mu_{mn}cos\theta t)f_{mn}=0\,,\quad \mu_{mn}=\frac{pt}{2p_{mn}^*} \tag{73}$$

Here ω_{mn} is determined by formulae (62), (63), and p_{mn}^* is determined by formulae (66) and (67).

Such values of frequencies, in case of which unstability takes place, are obtained.

In the sixth paragraph the problem of plate bent, taking into account the shares, is studied. The case, when the external normal load changes only in one direction, is considered. We shall have $D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}=q(y)$ (74)

$$w=\frac{\partial^2 w}{\partial y^2}=0\,,\quad \text{when } y=0 \text{ and } y=b \tag{75}$$

The solution of equation (74) is searched

$$w=w_0(y)+w_1(x,y) \tag{76}$$

in the form, where w_0 satisfies equation

$$D_{22}\frac{\partial^4 w_0}{\partial y^4}=q(y) \tag{77}$$

and $w_0(0)=w_0''(0)=w_0(b)=w_0''(b)=0$ conditions, and $w_1(x,y)$ - equation

$$D_{11}\frac{\partial^4 w_1}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w_1}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w_1}{\partial y^4}=0 \tag{78}$$

in case of boundary conditions $w_0(y)$ will have the following form

$$w_0 = \frac{b^2}{D_{22}\pi^4}\Sigma_{n=1}^{\infty}\frac{a_n}{n^4} \, sin\mu_n y \, , \quad \left(\mu_n = \frac{n\pi}{b}\right) \tag{79}$$

The unknow function $w_1(x,y)$ will be searched in the following form`

$$w_1 = \Sigma_{n=1}^{\infty} X_n(x) sin\mu_n y \tag{80}$$

and for the determination of the unkown function $X_n(x)$ we get differential equation

$$D_{11}X_n^{IV} - 2(D_{12} + 2D_{66})\mu_n^2 X_n'' + D_{22}\mu_n^4 X_n = 0 \tag{81}$$

the characteristic equation of which may have roots of three types. Particularly, when the roots are real and are equal to each other k, $(k > 0)$, then the corresponding solution $X_n(x)$ is equal to

$$X_n(x) = (A_n + B_n x)ch\mu_n kx + (C_n + D_n x)sh\mu_n kx \tag{82}$$

For the bent we shall have the following expression

$$w_1 = \Sigma_{n=1}^{\infty}\left(\frac{a_n b^2}{D_{22}\pi^4 n^4} + (A_n + B_n x)ch\mu_n kx + (C_n + D_n x)sh\mu_n kx\right) sin\mu_n y \tag{83}$$

Having expression w, it is possible to obtain the expressions of the rest of the physical values.

Some private cases are considered. In case of different boundary conditions, the formulae for the calculation of the bents are obtained.

In the fourth chapter multilayered plates theory general provisions are described, formulae, which are necessary for the solution of plates bent, obligatory vibration and stability problems, are derived. Problems of optimal management, free vibrations and layered composite plates stability made on the base of fiber filler are considered. Optimal management problem for the plate bending vibrations under the action of the changing load, applied on the plate surface made of multilayered composite material is studied. A numerical example is considered, the results of the solution are illustrated in the form of graphs.

In the first paragraph for the calculations of the multilayered anisotropic composite plate, made of fibers, the formulae, which are necessary for the solution of plate bending, obligatory vibrations, stability problems, are derived. It is assumed that all the layers of the plate are of the same h thickness and have the same properties. The general thickness of the plate is equal to $H = 2Nh$. The fibers of each layer are in the plane parallel to XOY coordinate plane and towards the axis OX they make φ_k angle. The direction of the layers fibers towards OX axis can be directed towards XOY plane symmetric or nonsymmetric.

The equation of a multilayered rectangular plate obligatory vibration in case of orthotropic materials is described by the equation

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2D_3\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{22}\frac{\partial^4 w}{\partial x^4} + p\frac{\partial^2 w}{\partial x^2} + 2Nh\rho\frac{\partial^2 w}{\partial t^2} + Q(x,y,t) = 0 \tag{84}$$

and

$$w = w_0(x,y), \; \frac{\partial w}{\partial t} = v_0(x.y), \text{ when } t = 0, \tag{85}$$

by initial conditions. Here p is the longitudinal force, acting on the medium surface (plane), ρ is the thickness of the material, $Q\,(x,y,t)$ is the transverse changing force.

A system, consisting of two equations, with the plates fibers directions, with unknown φ_k angles, in case of which the collateral rigidities of the plate anisotropic layers D_{16} and D_{26} will be equal to zero. Certain problems, when the fibers are made of bioplast, the number of layers is equal to four, six, eight, are conserved.

When $N=1$, then the plate consists of only two layers with $2h$ thickness and in φ_1 direction of the fibers.

For the plates consisting of four layers system $(N=2)$ consists of two homogeneous equations.

$$sin2\varphi_1 + 7sin2\varphi_2 = 0$$

$$sin4\varphi_1 + 7sin4\varphi_2 = 0, \tag{86}$$

Which has the following four solutions.

a) Zero solution $\varphi_1 = \varphi_2 = 0$, b) $\varphi_1 = 0$, $\varphi_2 = \pi/2$,

c) $\varphi_2 = 0$, $\varphi_1 = \pi/2$, d) the angles are equal to $\varphi_1 = \varphi_2 = \pi/2$:

In the same way the case of the plates, consisting of six $(N=3)$ and eight $(N=4)$ layers have been considered.

In the second paragraph multilayered composite plates free bending vibrations, which are described by equation, (84) when $p = 0$ and $Q(x,y,t) = 0$:

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2D_3\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial x^4} + 2N\rho h\frac{\partial^2 w}{\partial t^2} = 0 \tag{87}$$

the soloution of equation (87) is searched in the following form

$$w = (A\cos\omega t + B\sin\omega t)W(x,y) \tag{88}$$

Putting (88) into (87) for, $W(x,y)$ we shall have the following equation

$$D_{11}\frac{\partial^4 W}{\partial x^4} + 2D_3\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 W}{\partial x^4} - \omega^2 2N\rho hW = 0 \tag{89}$$

For the study of the free vibrations question, it is necessary to solve this equation in case of certain conditions. In case of a fastened, supported or free edge for W the boundary conditions do not differ from the conditions for w. The requirement, that function W should satisfy the boundary conditions and be the solution of equation (89), brings to homogeneous equations system for the unknown constants.

Let the plate bending vibrations W_{mn} private function satisfy equation (89) and the plate free support boundary conditions (2).

The solution of equation (89) is searched in the following form

$$W_{mn} = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{90}$$

We shall have

$$D_{11}\left(\frac{m\pi}{a}\right)^4 + 2D_3\left(\frac{mn\pi^2}{ab}\right)^2 + D_{22}\left(\frac{n\pi}{b}\right)^4 - \omega_{mn}^2 2nh\rho = 0$$

$$\omega_{mn} = \frac{\pi^2}{b^2}\sqrt{\frac{1}{2nh\rho}}\sqrt{D_{11}\left(\frac{m}{c}\right)^4 + 2D_3 n^2\left(\frac{m}{c}\right)^2 + D_{22}n^4}, \quad c = a/b: \tag{91}$$

The frequency of the basic mode is determined by the same equation, when $m=1$ and $n=1$

$$\omega_{11} = \frac{\pi^2}{a^2}\sqrt{\frac{1}{2nh\rho}}\sqrt{(D_{11} + 2D_3c^2 + D_{22}c^4)} \tag{92}$$

Particulary, for a square plate $(b = a, c = 1)$

$$\omega_{11} = \frac{\pi^2}{a^2}\sqrt{\frac{1}{2nh\rho}}\sqrt{D_{11} + 2D_3 + D_{22}} \tag{93}$$

The plate layers D_{11}, D_{22}, D_3 rigidities depend on the direction of the fibers classification in the plate layers.

Composite plates transverse vibrations frequency characteristics, depending on the relation of the plate sides and the direction of the fibers classification in its layers, when the fiber is made of boroplast, the number of the layers is equal to four, six and eight, are studied. The results are illustrated in the form of graphs.

In the third paragraph the stability problem of a layered plate, consisting of composite layers, is considered and this problem, in case of the orthotropic plate $(D_{16} = D_{26} = 0)$, is described with equation (87), when $\rho = 0$, $Q(x, y, t) = 0$. It has the following form

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2D_3\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} + p\frac{\partial^2 w}{\partial x^2} = 0 \tag{94}$$

The plate sides are highly fastened, the boundary conditions are described by equations (2). The solution of equation (94), satisfying condition (2) is searched in the form of

$$W_{mn} = A_{mn} sin\frac{m\pi x}{a} sin\frac{n\pi y}{b} \tag{95}$$

where A_{mn} are the constant coefficients, and m, n are the constant numbers. Putting (95) into (94) we get

$$A_{mn}\left\{\pi^4\left[D_{11}\left(\frac{m}{a}\right)^4 + 2D_3\left(\frac{mn}{ab}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4\right] - p\pi^2\left(\frac{m}{a}\right)^2\right\} = 0 \tag{96}$$

Non-zero solution is obtained by equalizing the expression in the figured bracket to zero. From here we find.

$$p = \frac{\pi^2\sqrt{D_{11}D_{22}}}{b^2}\left[\sqrt{\frac{D_{11}}{D_{22}}}\left(\frac{m}{c}\right)^2 + \frac{2D_3}{\sqrt{D_{11}D_{22}}}n^2 + \sqrt{\frac{D_{22}}{D_{11}}}\left(\frac{c}{m}\right)^2 n^4\right] \tag{97}$$

When $n = 1$, we obtain the well-known solutions

$$p_{кр} = \frac{\pi^2\sqrt{D_{11}D_{22}}}{b^2}\left[\sqrt{\frac{D_{11}}{D_{22}}}\left(\frac{m}{c}\right)^2 + \frac{2D_3}{\sqrt{D_{11}D_{22}}} + \sqrt{\frac{D_{22}}{D_{11}}}\left(\frac{c}{m}\right)^2\right] \tag{98}$$

or

$$p_{кр} = \frac{\pi^2\sqrt{D_{11}D_{22}}}{b^2}K_\sigma, \; K_\sigma = \left[\sqrt{\frac{D_{11}}{D_{22}}}\left(\frac{m}{c}\right)^2 + \frac{2D_3}{\sqrt{D_{11}D_{22}}} + \sqrt{\frac{D_{22}}{D_{11}}}\left(\frac{c}{m}\right)^2\right]$$

When $D_{11} = D_{22} = D_3 = D$, that is, the plate is made of isotropic material, from (98) we shall have

$$p_{\text{кр}} = \frac{\pi^2 D}{b^2}\left(\frac{m}{c} + \frac{c}{m}\right)^2, \tag{99}$$

which corresponds to the well-known formula.

The critical forces, depending on the sides relationship and the layers fiber direction, in case of which the plane balanced state looses its stability, are determined.

Certain examples, when the fiber is of boroplast, the number of the layers is equal to four, six and eight, are considered. The results are illustrated in the form of a graphs.

In the fourth paragraph we consider optimal management of a plate made of multilayerd composite material for the plate bending vibrations under the action of the changing load normally applied on the plate plane.

The differential on the plate of the anisotropic plate forced vibrations in case of the transverase load zero value is given by equation

$$\frac{1}{\rho h}\left(D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4}\right) +$$
$$+\frac{\partial^2 w}{\partial t^2} = \frac{1}{\rho h}Q(x, y, t) \tag{100}$$

If $D_{16} = D_{26} = 0$, than equation (100) admits the form

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} + \rho h\frac{\partial^2 w}{\partial t^2} = Q(x, y, t) \tag{101}$$

The plate sides are hingely fastened. The initial conditions are

$$w = F_1(x, y), \quad \frac{\partial w}{\partial t} = F_2(x, y), \text{ when } t = 0 \quad (t_0 = 0) \tag{102}$$

The management problem lies in finding such a solution, in the case of which at $t = t_1$ moment of time, the displacement of the plate medium surface and the displacement velocity are brought to a new state with $\Phi_1(x, y)$ and $\Phi_2(x, y)$ functions.

$$w(x, y, t) = \Phi_1(x, y), \quad \frac{\partial w(x,y,t)}{\partial t} = \Phi_2(x, y) \text{ when } \quad t = t_1 \tag{103}$$

a requirement should be added to this condition

$$J = \int_0^a \int_0^b \int_0^{t_1} Q^2\, dx\, dy\, dt \tag{104}$$

In order to have minimal value $J = J_{min}$ for the functional extreme

The solution of the equation will be represented in the following form

$$w(x, y, t) = \sum_{m,n=1}^{\infty} f_{mn}(t) sin\lambda_m x\, sin\mu_n y, \quad \lambda_m = \pi nx/a, \quad \mu_n = \pi nx/b \tag{105}$$

In order to find the unknown functions $f_{mn}(t)$, in the left part of equation (101) we install (105), and in the right part $Q(x, y, t)$

$$Q = \sum_{m,n=1}^{\infty} q_{mn}(t) sin\lambda_m x\, sin\mu_n y \tag{106}$$

Fourier series of function

$$\frac{d^2 f_{mn}(t)}{dt^2} + \omega^2 f_{mn}(t) = \frac{1}{\rho h} q_{mn}(t) \tag{107}$$

As a result we have differential equation where

$$\omega_{mn}^2 = \frac{1}{\rho h}\left(D_{11}\lambda_m^4 + 2(D_{12} + 2D_{66})\lambda_m^2 \mu_n^2 + D_{22}\mu_n^4\right) \tag{108}$$

The solution of equation (107) has got the following form.

$$f_{mn}(t) = A_{mn}cos\omega_{mn}t + B_{mn}sin\omega_{mn}t + \frac{1}{\omega_{mn}}\int_0^t q_{mn}(\tau)\, sin\omega_{mn}(t-\tau)d\tau \quad (109)$$

The transverse forces, bringing the plate from the initial state to the final state at the denoted time, are found. On the load an additional requirement is set - minimization of the functional, depending on its value square. The problem is solved by Fourier method.

As a final result we obtain

$$w(x,y,t) = \sum_{m=1,n=1}^{\infty} (a_{mn}\, cos\omega t + \frac{b_{mn}}{\omega} sin\omega t + \frac{1}{4h\rho\omega^2}(\beta_{mn}\omega t cos\omega t -$$

$$-(\beta_{mn} + \alpha_{mn}\omega t sin\omega t)))sin\frac{m\pi}{a}x\, sin\frac{m\pi}{b}y \quad (110)$$

$$w_t(x,y,t) = \sum_{m=1,n=1}^{\infty} (-a_{mn}\,\omega sin\omega t + b_{mn} cos\omega t + \frac{1}{4h\rho\omega^2}(\beta_{mn}\omega cos\omega t -$$

$$-\omega(\beta_{mn} + \alpha_{mn}\omega t sin\omega t) - \omega(\alpha_{mn} + \beta_{mn}\omega t)sin\omega t)))sin\frac{m\pi}{a}x\, sin\frac{m\pi}{b}y$$

Numerical calculation is fulfilled, too.

Conclusion. The thesis is devoted to one of the modern themes of mechanics of deformable solid body- to the thin-walled constructions stability, vibration, and optimal design problems, which have both theoretical and applied great significance. In the thesis, particularly, the following result is obtained.

1. The author has solved one-layered rectangular plate stability problem in case of the arbitrarily distributed forces, composite viscoelastic multilayered rectangular plate stability problem towards the coordinate plane in case of the the layers different arrangement, when on two sides of the plate uniformly compressing forces and two layered plate stability problem taking into account the transverse shear, act.

2. Symmetric three-layered anisotropic inhomogeneous plate optimal design problem is solved. In the fixed case of the plate free vibrations basic frequency we found such plate thickness, in case of which the mass obtains the smallest value. Non - linear equations system, which is solved by numberical method, using Matlab program, is obtained.

3. Cylindrical shell stability problem, in case of external axis-symmetric arbitrarily normal load, freely supported edge and “Floating fastening” boundary conditions is solved. The stability critical paramenters are determined.

4. Optimal design problems of a homogneous anisotropic one-layered cylindrical shell of changing thickness, one-layered homogeneous orthotropic torsion shell of changing thickness and open cylindrical shell are solved. Such cylinder thickness, in case of which the shell mass obtains the smallest value, is found.

5. The optimal design problem of a bar, having a circular changing cross-section, is solved: to find the cross-section changing cut characterizing such a law, when the biggest bending of the bar gets the smallest value, and the bar volume obtains the beforehand given value. Formulae of rod bent and optimal thickness are obtained.

6. Plate bent, vibrations and stability problems, taking into account the transverse shares, are solved. It is shown, that, taking into account the shears, plate vibration frequency and the critical load value are small from the determined by classical theory.

7. The plate dynamic stability problem, when the force, acting on its plane is a periodical function depending on time. Such values of frequencies, which cause unstability, are obtained.

8. Bent, obligatory vibration, stability and optimal management problems of the plate, made of multilayered composite material, are solved. In the vibration problem the basic frequency behavior, depending on the relationship of its sides and the direction of the fibers, is studied. The critical forces are determined. The transverse forces which bring the plate from the initial state to the final state in the denoted time, are found. For the considered problems numerical calculations are done, the results of the calculations are illustrated in the form of graphs.

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Եզրակացություն

Ատենախոսությունը նվիրված է դեֆորմացվող պինդ մարմնի մեխանիկայի արդիական թեմաներից մեկին՝ բարակապատ կառույցների կայունության, տատանումների և օպտիմալ նախագծման խնդիրներին, որոնք ունեն թե՛ տեսական, թե՛ կիրառական մեծ նշանակություն: Ատենախոսության մեջ, մասնավորապես, ստացված են հետևյալ արդյունքները.

1. Լուծված են միաշերտ ուղղանկյուն սալի կայունության խնդիրը կամայական բաշխված ճիգերի դեպքում, կոմպոզիցիոն առաձգամածուցիկ բազմաշերտ ուղղանկյուն սալի կայնության խնդիրը կոորդինատական հարթության նկատմամբ շերտերի տարբեր դասավորվածության դեպքում, երբ սալի երկու կողմերի վրա ազդում են հավասարաչափ սեղմող ճիգեր և շերտավոր սալի կայնության երկու խնդիրներ՝ ընդլայնական սահքերի հաշվառումով:

2. Լուծված է սիմետրիկ եռաշերտ անիզոտրոպ անհամասեռ սալի օպտիմալ նախագծման խնդիր: Սալի ազատ տատանումների հիմնական հաճախության սևեռված արժեքի դեպքեւմ գտնվել է սալի այնպիսի հաստություն, որի դեպքում սալի զանգվածը ստանում է ամենափոքր արժեք: Ստացված է ոչ գծային հավասարումների համակարգ, որը լուծված է թվային մեթոդով, օգտագործելով Մաթլաբ ծրագիրը:

3. Լուծված է գլանային թաղանթի կայունության խնդիրը արտաքին առանցքա-սիմետրիկ կամայական նորմալ բեռի դեպքում, ազատ հենված եզրային և «լողող ամրակցում» եզրային պայմանների դեպքերում: Որոշված են կայունության կրիտի-կական պարամետրերը:

4. Լուծված են փոփոխական հաստության համասեռ անիզոտրոպ միաշերտ գլանային թաղանթի, միաշերտ փոփոխական հաստության համասեռ օրթոտրոպ պտտման թաղանթի և բաց գլանային թաղանթի օպտիմալ նախագծման խնդիրները: Գտնված է թաղանթի այնպիսի հաստություն, որի դեպքում թաղանթի զանգվածը ստանում է ամենափոքր արժեք:

5. Լուծված է շրջանային փոփոխական լայնական հատույթ ունեցող ձողի օպտիմալ նախագծման խնդիր. գտնել լայնական հատույթի փոփոխական կտրվածքը բնութագրող այնպիսի օրենք, որ ձողի ամենամեծ ճկվածքը ստանա փոքրագույն արժեք, իսկ ձողի ծավալն ընդունի նախապես տրված արժեք: Ստացված են ձողի ճկվածքի և օպտիմալ հաստության բանաձևեր:

6. Լուծված են սալի ծռման, տատանումների և կայունության խնդիրներ ընդլայնական սահքերի հաշվառումով: Ցույց է տրված, որ սահքերով հաշվառած սալի տատանման հաճախությունը և կրիտիկական բեռի մեծությունը փոքր են դասական տեսությամբ որոշվածից:

7. Ուսումնասիրված է սալի դինամիկական կայունության խնդիրը, երբ նրա հարթությունում ազդող ճիգը ժամանակից պարբերական ֆունկցիա է: Ստացված են հաճախությունների այնպիսի արժեքներ, որոնց դեպքում տեղի ունի անկայունություն:

8. Լուծված են բազմաշերտ կոմպոզիցիոն նյութից պատրաստված սալերի ծռման, հարկադրական տատանման, կայունության և օպտիմալ կառավարման խնդիրներ: Տատանման խնդրում ուսումնասիրված է հիմնական հաճախաության վարքը

կախված նրա կողմերի հարաբերությունից և շերտերում մանրաթելի ուղղությունից: Որոշված են կրիտիկական ուժերը: Գտնված են նշված ժամանակահատվածում սալը սկզբնական վիճակից վերջնական վիճակին բերող լայնական ուժերը: Դիտարկված խնդիրների համար կատարված են թվային հաշվարկներ, հաշվարկների արդյունքները պատկերված են գրաֆիկների տեսքով:

РЕЗЮМЕ

диссертации Дарядар Мохаммад Халегвердиевич «Задачи устойчивости, колебаний и оптимального проектирования многослойных пластин и оболочек»

Диссертация посвящена одной из актуальных тем механики деформируемого твердого тела: устойчивости, колебаниям и оптимальному проектированию тонко-стенных конструкций, имеющей как теоретическое, так и прикладное значение. В диссертации, в частности, получены следующие результаты:

1. Решены задача устойчивости однослойной прямоугольной плиты при произвольных распределенных усилиях, задача устойчивости композиционной вязкоупругой многослойной прямоугольной плиты в случае разного расположения слоев относительно координатной плоскости, когда на обе стороны плиты действуют равномерно распределенные сжимающие усилия, и две задачи устойчивости слоистой плиты с учетом поперечных сдвигов.
2. Решена задача оптимального проектирования симметричной трехслойной анизотропной, неоднородной пластины. При определенном значении основной частоты свободных колебаний пластины найдена такая толщина, при которой масса пластины оказывается наименьшей. Получена система нелинейных уравнений, которая решается численно при помощи программы Матлаб.
3. Решена задача устойчивости цилиндрической оболочки в случае произвольной осесимметричной нормальной внешней нагрузки, при граничных условиях свободного опирания и скользящей заделки. Определены критические параметры устойчивости.
4. Решены задачи оптимального проектирования однородной однослойной анизотропной цилиндрической оболочки переменной толщины, однородной однослойной ортотропной оболочки вращения переменной толщины и открытой цилиндрической оболочки. Найдена толщина оболочки, в случае которой ее масса имеет наименьшее значение.
5. Решена задача оптимального проектирования круглого стержня переменной толщины: найти закон характеризуюее переменное поперечное сечение стержня так, чтобы максимальный прогиб стержня получал наименьшее значение, а объём стержня принимало заранее заданное значение. Получены формулы для прогиба стержня и для оптимальной толщины.
6. Решены задачи на изгиб, колебания и устойчивость для платинки с учетом поперечных сдвигов. Показано, что частота колебаний платинки и величина критической силы с учетом поперечных сдвигов меньше от тех, которые получены по классической теории.
7. Изучена проблема динамической устойчивости пластинки, когда сила, дей-ствующая на её плоскость, является периодической функцией времени. Получены значения частоты, при которыхимеется неустойчивость.

8. Решены задачи на изгиб, устойчивость, вынужденные колебания и оптимальное проектирование для многослойной пластины из композиционного материала. В задаче колебания исследовано поведение основной частоты в зависимости от отношения ее сторон и от направления волокон слоев. Определены критические силы. Найдены поперечные силы, которые в заданном промежутке времени из начального состояния плиты приводят в окончательное состояние. В рассмотренных задачах сделаны численные расчеты, результаты расчетов представлены в виде графиков.