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(ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ)

Ապրեսյան Ելենա Անդրանիկի

ՀՈԼԼԻ ՔՎԱՆՏԱՅԻՆ ԵՐԵՎՈՒՅԹԻ ՈՐՈՇ ԱՍՊԵԿՏՆԵՐԸ

Ա.04.02 - «Տեսական ֆիզիկա» մասնագիտությամբ ֆիզիկամաթեմատիկական գիտությունների թեկնածուի գիտական աստիճանի հայցման ատենախոսության

ՍԵՂՄԱԳԻՐ

ԵՐԵՎԱՆ – 2019

A.I.ALIKHANYAN NATIONAL SCIENCE LABORATORY
(YEREVAN PHYSICS INSTITUTE)

Apresyan Yelena

SOME ASPECTS OF QUANTUM HALL EFFECT

SYNOPSIS

of Dissertation in 01.04.02 – Theoretical Physics Presented for the degree of candidate in physical and mathematical Sciences

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Ատենախոսության թեման հաստատվել է Ա. Ի. Ալիխանյանի անվան Ազգային Գիտական Լաբորատորիայի (ԵրՖի) գիտական խորհուրդում:
Գիտական ղեկավար՝

Ֆիզ.-մաթ գիտ. դոկտոր Ա.Սեդրակյան(ԱԱԳԼ)

Պաշտոնական ընդդիմախոսներ՝

Ֆիզ.-մաթ գիտ. դոկտոր Ռ.Պողոսյան(ԱԱԳԼ)

Ֆիզ.-մաթ գիտ. դոկտոր Դ.Սահակյան(ԱԱԳԼ)

Առաջատար կազմակերպություն՝ Երևանի Պետական Համալսարան
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Ֆիզ.-մաթ գիտ. դոկտոր Հ. Մարուքյան

The subject of the dissertation is approved by the scientific council of the A.Alikhanyan National Science Laboratory (AANL)
Scientific advisor: d.ph-math sci. A.Sedrakyan (AANL)
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Defense takes place on 15^{th} of March at 14:00 on meeting of the special council of the Higher Attestation Commision of RA 024 ”Physics” , acting in A.Alikhanyan National science Laboratory(Yerevan-0036, st. Alikhanyan Brothers 2)
The dissertation can be learnt at the AANL library
The synopsis is sent out on 29.01.2019.
Scientific secretary of the special council
Doctor of physical and mathematical sciences H.Marukyan

Introduction

The quantum Hall effect was a constant source of new ideas, most of which are related to how topology invades quantum physics. Attractive examples include topological insulator, topological order, and topological quantum computations. Basically, all of these phenomena are an impressive theoretical construct that involves traveling through some of the most fascinating and important developments in the field of theoretical and mathematical physics in recent decades.

The first attack on the problem focussed on the microscopic aspects of the electron wavefunctions. Graphene now is attracting scientists with its peculiar material characteristics.Electrons in graphene strongly interact and exhibit fractional quantum Hall effect. But it is remarkable that evidence of the collective behavior of electrons in graphene is still lacking.

The integer quantum Hall effect can be described only in terms of individual electrons in a magnetic field while the fractional quantum Hall effect can be understood by studying the collective behaviour of all the electrons.The quantum Hall effect is also studied in context of conform field theory. Recently, it has been proposed that the order parameter of the fractional quantum Hall effect is related to the vertex operator, and the ground state wave function of a certain fractional filling factor can be expressed in terms of the N-point correlation function of vertex operator.

The application of conformal field theory has thus been extended into a rather specific condensed matter phenomenon. The Laughlin states for N interacting electrons at the plateaus of the fractional Hall effect are examined in the thermodynamic limit of large N.

The quantum Hall systems is a major paradigm in condensed matter physics, with important applications such as resistance metrology and measurements of fundamental constants. In the recent years, it has been shown that the quantum Hall effect is just one member of a much larger family of topologically specific quantum states, some of which contain the quantum spin Hall effect which is also known as the 2D topological insulator and 3D TI's.

The main task of the thesis is to study some aspects of quantum Hall effect and the relation with Liouville field theory.

Timeliness and relevance

Graphene is now attracting scientists with its peculiar material characteristics. Electrons in graphene strongly interact and therefore exhibit fractional quantum Hall effect. But remarkably, the evidence for collective behaviour of electrons in graphene still is absent. The integer quantum Hall effect can be described only in terms of individual electrons in a magnetic field while the fractional Hall effect is related to the collective behavior of electrons.

The quantum Hall effect is also studied in the context of conformal field theory. Two-dimensional conformal field theories describe statistical systems at critical points and provide the classical solutions of string theory. Recently, it has been proposed that the order parameter of the fractional quantum Hall effect is related to the vertex operator, and the ground state wave function of a certain fractional filling factor can be expressed in terms of the N-point correlation function of vertex operators. It's important to notice that the Fractional Quantum Hall effect is possible also to describe by the Liuoville field theory, whose cosmological constant should play a role of a chemical potential, which plays an important role in quantum Hall effect.

The electrical conductivity of graphene, a two-dimensional hexagonal lattice of carbon atoms, possesses many qualities necessary for promising applications in both fundamental physics and nanotechnology. At energies below a few electron volts the electronic properties of graphene are perfectly described by the Dirac model.

In a series of papers authors had calculated conductivity with non-zero gap, chemical potential,scattering rate and magnetic field. However current response functions were not studied properly in the presence of non-quantized external magnetic field and chemical potential.

The results of dissertation are important from point of view condensed matter physics and conformal field theory.

Aim of the dissertation

- To calculate the current-current correlation function of 3D massive Dirac fermions in the presence of chemical potential
- To study the current-current correlation function in presence of external non-quantized magnetic field and chemical potential
- To examine transport properties of topological insulator and analyze polarization operator
- To investigate the boundary Liouvile three point function in mini-superspace limit

- To explore topological defects in Liouville field theory with different cosmological constants which can play the role of chemical potential in quantum Hall effect

Novelty of the work

The basic meaning of the quantum Hall effects (integer and fractional) is that they are examples of physical systems in which quantization effects appear macroscopically.

This results arose from an interesting interplay between disorder, topology and interactions. The important factor is that an electron is in the magnetic field.
After solving the Schrödinger equation, so-called Landau levels are obtained, which have quantized energy values separated by a large gap. Actually, this is nothing special, since most quantum-mechanical problems have a discrete spectrum. But in the case of the quantum Hall effect, this discontinuity occurs when we measure the Hall conductivity. For the value of conductivity there are two possibilities that lead to integer quantum Hall and fractional quantum Hall effects.

The significance of quantum Hall effect research is related to modern technologies. There are some classes of materials whose electric conductivity increases with temperature and whose charge carriers can be either positive or negative, depending on the impurity introduced into them these materials are called semiconductors.
The conductivity of fermions in graphene in most general situation was investigated in a large amount of papers. However, as it appears, current response functions were not studied properly. In this work we have calculated current-current correlation function in one loop approximation and in a presence of non-zero chemical potential and external magnetic field. In modern physics there are materials, such as topological insulators (TI) with edge states, Bose-Einstein condensates of Rb atoms with spin-orbit interactions and honeycomb lattices with next to nearest neighbor (NNN) interactions,where the spectrum of non-relativistic particles combined with relativistic Dirac component.In work we present polarization operator of non-relativistic fermions with spin-orbit (SO)Rashba interaction. The spectrum of this fermions is moat type having minimum on a circle. Contrary to Dirac or non-relativistic fermions Fermi sea here has a geometry of Corbino disk which reflects in a transport properties of excitations.
Recently the various semiclassical limits of the Liouville correlation functions appeared in different instances.For example we can mention study of conformal blocks in AdS/CFT correspondence, semiclassical limits of the Nekrasov partition functions, minisuperspace limit of correlation functions , semiclassical limit of correlation functions in the presence of defects and boundaries and the most recently found application of the semiclasical limit of Liouville field theory to the SYK

problem.We study mini-superspace semi-classical limit of the boundary three-point function in the Liouville field theory.We compute also matrix elements for the Morse potential.An exact agreement between the former and the latter is found.We show that both of them are given by the generalized hypergeometric functions.

Topological defects in the Louville field theory with the same cosmological constants on the both side were constructed. Two-point functions in the presence of defects were computed using the Cardy-Lewellen equation for defects. It was derived that there exist two families of defects, discrete, with one-dimensional world-volume, and continuous, with two-dimensional world-volume. In this work we generalize above mentioned calculations to the case of the different cosmological constants.We construct topological defects in the Liouville field theory producing jump in the value of cosmological constant. We construct it using the Cardy-Lewellen equation for the two-point function with defect. We show that there are continuous and discrete families of such kind of defects. For the continuous family of defects we also find the Lagrangian description and check its agreement with the solution of the Cardy-Lewellen equation using the heavy asymptotic semiclasscial limit.

Practical value

- Calculating the response function we will have information about conductivity, the latter plays important role in observation of quantum Hall effect.
- The behavior of conductivity can help to observe transport properties of materials.
- The result obtained for topological insulators has practical value for quantum computers.
- The cosmological constant presented in 2D Liouville field theories can play the role of chemical potential in condensed matter physics.

Main points to defend

In the dissertation

- we will represent the expression of current-current correlation function in second order of Feynman diagram with chemical potential,
- for third order Feynamn’s diagram the current response function will be calculated in the presence of chemical potential and magnetic field,
- for topological insulator with moat spectra we will calculate and analyze the operator of polarization.

- the boundary three-point function in the BLFT will be computed in and expressed via double Gamma and double Sine functions,
- using known asymptotic properties of the double Gamma and Sine functions we will show that in the mini-superspace limit the boundary three-point function can be expressed via the Meijer functions with the unit argument or equivalently via the generalized hypergeometric functions with the unit argument,
- also we will construct topological defects gluing 2D Liouville field theories with different cosmological constants.

Structure of the dissertation

The dissertation consists of the introduction, three chapters and finally the list of used literature and figures.

The complete list of **used literature** is presented in the thesis.

Dissertation's presentation

The main results of the thesis were discussed at the YerPhI Joint Theoretical Physics Laboratory and Yerevan State University(YSU) and 2018 joint FAR/ANSEF-ICTP and RDP-VW summer school in theoretical physics .

Content of the dissertation

In **introduction** we briefly discuss the graphene, some aspects of quantum Hall effect, topological insulators and boundary Liouville field theory with different cosmological constants. We describe the equation of motion in external magnetic field. The fact that magnetic field causes charged particles to move in circles creates the Hall effect in context of Drude model.

We also look at the quantum mechanics of free particles moving against the background of a magnetic field and creating Landau level. In the presence of a magnetic field, the energy levels of the particles become equal from each other, where the gap between each level is proportional to the magnetic field. These energy levels are called Landau levels.

The **first** chapter shows that the reaction of the fermion system to external gauge fields is determined by the current-current correlation function.Transport properties of different physical quantities are determined by zero energy-momentum limit of it. It is well known close to half-filling the physics of graphene is described by $(2+1)$ dimensional Dirac theory.

In this chapter we calculate the current-current correlation function in Dirac theory in the presence of chemical potential η and gap m.

We intend to calculate the current-current correlation function for the three-dimensional theory with the kinetic part for the fermions and the interaction term with gauge field in the one- loop approximation.

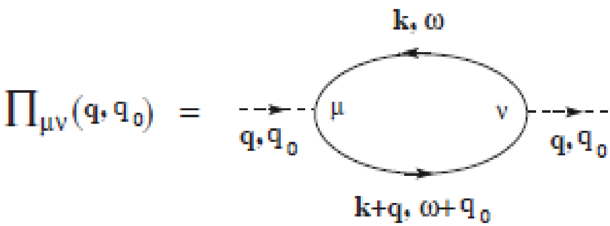


Fig.1. The lower order Feynman diagram for current-current correlation function.

The action which describes the graphene in the Effective Field Theory (EFT) framework via four-component massive Dirac fermions with instantaneous three-dimensional Coulomb interactions is the following (in Euclidean space time)

$$S_g = -\sum_{i=1}^{N_f} \int d^2x dt \bar{\psi}_i \left(\gamma^0 \partial_0 + v\gamma^k \partial_k + iA_0\gamma^0 + m\right)\psi_i + \frac{1}{2g^2}\int d^2x dt (\partial_k A_0)^2.$$

The current-current correlation function is defined by Feynman diagram (see Fig.1).

$$\Pi_{\mu\nu}(\mathbf{r}-\mathbf{r}') = \langle j_\mu(\mathbf{r}) j_\nu(\mathbf{r}') \rangle$$

and in momentum space it reads

$$\Pi_{\mu\nu}=\int_{-\infty}^{\infty}\frac{d^3k}{(2\pi)^3}Tr\left[\sigma_\mu G(k)\sigma_\nu G(k+q)\right]=\int_{-\infty}^{\infty}\frac{d^3k}{(2\pi)^3}\frac{Tr\left[\sigma_\mu(k_\rho\sigma_\rho-m)\sigma_\nu(k_\lambda\sigma_\lambda-m)\right]}{\left[k^2+m^2\right]\left[(k+q)^2+m^2\right]}.$$

The four-component fermionic structure is conditioned by the existence of the quasi-particle excitations in two sublattices in the graphene around two Dirac points.
Note that for graphene model's case with four-component fermions, the last term linear by mass must be annihilated due to the contributions of two different two-dimensional fermions with opposite parities. The expression of Trace combined with the formula of Feynman parametrization. The shift of integration energy/momentum gives

$$\Pi_{\mu\nu}=2\int_{\infty}^{\infty}\frac{d^3k}{(2\pi)^3}\int_0^1 dx\frac{2k^\rho k^\nu-\delta^{\rho\nu}\left(k^2+m^2+q^2x(1-x)\right)+2x(1-x)(\delta^{\rho\nu}q^2-q^\rho q^\nu)}{\left(k^2+m^2+q^2x(1-x)\right)^2}$$

$$-im\,\epsilon_{\mu\nu\rho}\,q_\rho\int_{\infty}^{\infty}\frac{d^3k}{(2\pi)^3}\int_0^1 dx\frac{1}{\left(k^2+m^2+q^2x(1-x)\right)^2}=\Pi_{\mu\nu}^{(1)}+\Pi_{\mu\nu}^{(2)}+\Pi_{\mu\nu}^{(3)}.$$

Thus, three terms are chosen in the last equation in this way: the first is the part that does not satisfy the condition of charge conservation .The third term is the anomaly part. Second, the main transversal term is

$$\Pi_{\mu\nu}^{(2)}=2(\delta^{\rho\nu}q^2-q^\rho q^\nu)\int_{-\infty}^{\infty}\frac{d^3k}{(2\pi)^3}\int_0^1 dx\frac{2x(1-x)}{\left(k^2+m^2+q^2x(1-x)\right)^2}$$

Then, for $q^2\geq 4(\eta^2-m^2)\geq 0$ when the square root in the expression of $x_{1,2}=\frac{1}{2}(1\pm\sqrt{1-\frac{4(\eta^2-m^2)}{q^2}})$ is real, the integral gives

$$\Pi_{\mu\nu}^{(2)}=\frac{(\delta^{\mu\nu}q^2-q^\mu q^\nu)}{\pi}\times\left[\frac{1}{12\eta}\left(1+\left(\frac{\eta^2+2m^2}{q^2}-1\right)\sqrt{1-\frac{4(\eta^2-m^2)}{q^2}}\right)+\frac{1-\frac{4m^2}{q^2}}{8q}\arctan\frac{q\sqrt{1-\frac{4(\eta^2-m^2)}{q^2}}}{2\eta}\right]$$

When $4(\eta^2-m^2)\geq q^2$ the integral becomes

$$\Pi^{(2)}_{\mu\nu}=\frac{(\delta^{\mu\nu}q^2-q^{\nu}q^{\nu})}{12\pi\eta}.$$

For the case $(\eta^2-m^2)\leq 0$ the expression for $x_{1,2}$ defines larger than segment $[0,1]$ region and we have to put $x_1=0, x_2=1$. We have a result

$$\Pi^{(2)}_{\mu\nu}=\frac{(\delta^{\mu\nu}q^2-q^{\mu}q^{\nu})}{\pi}\frac{1}{8q}\Big(\frac{2m}{q}+(1-\frac{4m^2}{q^2})\arctan\frac{q}{2m}\Big).$$

$$\Pi^{(3)}_{\mu\nu}=-\frac{im\,q_{\rho}\,\epsilon_{\mu\nu\rho}}{2\pi q}\arctan[\frac{q}{2|m|}]=-\frac{i}{4\pi}sign[m]\,q_{\rho}\,\epsilon_{\mu\nu\rho}+O(\frac{q^2}{m^2})$$

The result for the grapheme with two Dirac fields of opposite chirality is $\Pi_{\mu\nu_g}=2\Pi^{(2)}_{\mu\nu}$ because the anomaly term will be cancelled.

In this chapter we also calculate the response of fermionic system to external gauge fields in presence of magnetic field.The response function is determined by current-current correlation function. We study 2D dimensional Dirac electron system and calculate current-current correlation function in a presence of magnetic field B, chemical potential η and gap m.The magnetic field dependence of the current-current correlation function is defined by third order Feynman diagrams in Fig. 2. For current-current correlation function we get

$$\Pi_{\mu 3}=Ng^2\int_{-\infty}^{+\infty}\frac{d^3k}{(2\pi)^3}\mathrm{Tr}[\sigma_{\mu}G(\hat{k}^{+})A_{\rho}\sigma_{\rho}G(\hat{k}^{+}+\hat{p})\sigma_3 G(\hat{k}^{-})]=Ng^2\int_{-\infty}^{+\infty}\frac{d^3k}{(2\pi)^3}\mathrm{Tr}[\sigma_{\mu}G(\hat{k}^{+})A_{\rho}\sigma_{\rho}\frac{\hat{p}}{p^2+m^2}\sigma_3 G(\hat{k}^{-})]$$

where $G(\hat{k})=\dfrac{\hat{k}-m}{k^2+m^2}$, $k^{\pm}=(\vec{k}\pm\dfrac{\vec{q}}{2},\Omega\pm\dfrac{\omega}{2})$, $A_{\rho}\sigma_{\rho}\hat{p}=\vec{A}\vec{p}+i\epsilon_{\nu\rho}A_{\nu}p_{\rho}\sigma_3=iB\sigma_3$,

$k^2=\mathbf{k}^2+(\Omega+\Gamma+i\eta)$.

After calculation for response function is received

$$\Pi_{\mu 3}(B) = -\frac{iB}{4\pi|\eta|}\epsilon_{\mu\nu}q_{\nu}(\Gamma + i\eta)\Big(\frac{1}{m^2+\frac{q^2}{4}}\sqrt{1-\frac{4(\eta^2-m^2)}{q^2}}+\frac{1}{\eta^2}\Big(1-\sqrt{1-\frac{4(\eta^2-m^2)}{q^2}}\Big)\Big)$$

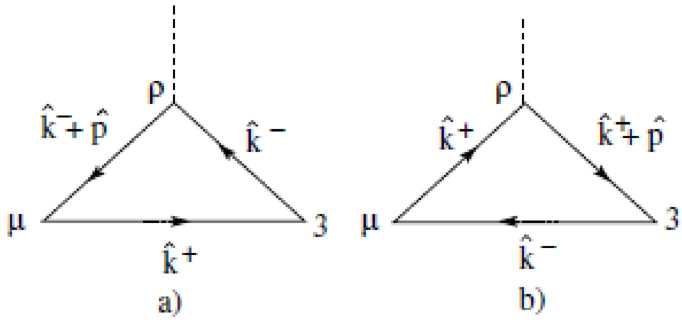


Fig.2. Third order Feynman diagram for current-current correlation function

The **second** chapter is about topological insulators. A topological insulator is a material with non-trivial symmetry-protected topological order that behaves as an insulator in its interior but whose surface contains conducting states, meaning that electrons can only move along the surface of the material. However, having a conducting surface is not unique to topological insulators, since ordinary band insulators can also support conductive surface states. What is special about topological insulators is that their surface states are symmetry-protected by particle number conservation and time reversal symmetry.The polarization operator is a mathematical construction that determines both the longitudinal and Hall conductivities on the one hand and the effective action of the gauge field defined by quantum fluctuations of fermions, on the other. In this chapter the calculation of the polarization operator with moat type spectrum is presented. The most common form of the main Hamiltonian of such systems is

$$H(k) = \epsilon_k + \sum_{i=x,y,z} d_i(k)\sigma_i$$

where $\epsilon_k = -\mu + D\vec{k}^2, \vec{d} = A\vec{k}, d_3 = \Delta - M\vec{k}^2$. For topological insulators $D<0$, while for cold atoms $D>0$. The edge states in TI or excitations on honeycomb lattice with NNN interaction come in pairs with states of opposite chirality defined by time reversal Hamiltonian

$$H^*(-\vec{k}) = \varepsilon_k + A(-k_x\sigma_x + k_y\sigma_y) + d_3\sigma_3^s$$

where $\sigma_3^s = s\sigma_3, s = \pm 1$ defines chirality.Without lost of generality we can take $\sigma_i^s = s\sigma_i$.The total Hamiltonian of such systems is

$$H = \begin{pmatrix} H(k) & 0 \\ 0, & H^*(-k) \end{pmatrix}$$

For action of fermions with particular chirality is

$$S = \bar{\psi}^+[\Omega - \epsilon_k - v_F \overrightarrow{\sigma^s}\vec{k} - d_3\sigma_3^s]\psi \cdot$$

Causal Green function can be written in a simple form

$$G(\Omega,\vec{k}) = \frac{1}{2}\left(\frac{1+\sigma^i n^i}{\Omega - E_k^- + i\eta_k^-} + \frac{1-\sigma^i n^i}{\Omega - E_k^+ + i\eta_k^+}\right)$$

where $n^i = k^i / k$ is the unite vector along momentum direction, $E^\pm = \varepsilon_k \pm \epsilon_k = D\vec{k}^2 \pm \sqrt{v_F{}^2\vec{k}^2 + d_3^2}, \eta_k^\pm = \eta sign(E_k^\pm - \mu)$ and μ is the chemical potential. For response function we get

$$\Pi_{\mu\nu}^{(1)}(k_{1F}) = \frac{1}{4}\int_0^{2\pi} \frac{d\phi}{(2\pi)^2} \frac{T_{\mu\nu}(k_{1F})\left[\vec{k}_{1F}\vec{q} - 2i\eta\dfrac{m\epsilon_{k_{1F}}}{\epsilon_{k_{1F}} - mv_F^2}\right]}{\omega + 2i\eta - \left(\dfrac{1}{m} - \dfrac{v_F^2}{\epsilon_k}\right)k_{1F}q\cos\phi}$$

$$= -\frac{T_{\mu\nu}(k_{1F},q)}{8\pi}\frac{1}{\left(\frac{1}{m}-\frac{v_F^2}{\epsilon_{k_{1F}}}\right)}\left(1-\frac{\omega}{\sqrt{(\omega+2i\eta)^2-\left(\frac{1}{m}-\frac{v_F^2}{\epsilon_{k_{1F}}}\right)^2 k_{1F}^2 q^2}}\right)+O\left(\frac{\omega^3}{\mu^3},\frac{q^3}{k_{1F}^3}\right)$$

where

$$T_{\mu\nu}(k_{1F},q) = 2\left(\frac{q_\mu q_\nu}{4}-\frac{k_{1F}^2}{2}\delta_{\mu\nu}+v_F^2 m^2\delta_{\mu\nu}\right)\frac{1}{m^2}\left(\frac{\Delta^2+v_F^2\overrightarrow{q}^2/4}{\epsilon_{k_{1F}}^2}\right)+4\left(\frac{k_{1F}^2}{2}\delta_{\mu\nu}-\frac{q_\mu q_\nu}{4}\right)\left(\frac{1}{m}+\frac{v_F^2}{\epsilon_k}\right)^2+2i\frac{v_F^2}{\epsilon_{k_{1F}}}\epsilon^{\mu\nu}\omega$$

Characteristic picture of two branches of this spectrum is presented in Fig.3, while only lower branch is forming the ground state.

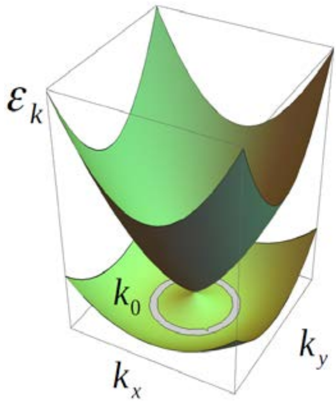


Fig.3. Two branches of spectra of moat type

In Fig.4a the characteristic form of the Fermi sea is presented .

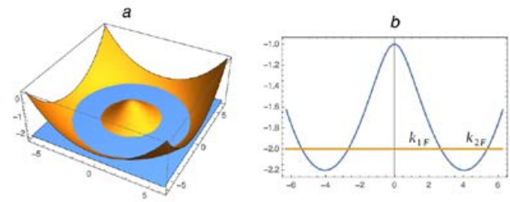
In result for longitudinal conductivity as a coefficient of linear response to external electric field is obtained

$$\sigma_{xx}(\omega) = \frac{T_{xx}(k_{1F},0)-T_{xx}(k_{2F},0)}{8\pi\left(\frac{1}{m}-\frac{v_F^2}{\epsilon_k}\right)}\frac{i}{\omega+2i\eta}$$

The expression shows,that at chemical potential $\mu = -\frac{m^2 v_F^4 + \Delta^2}{2mv_F^2}$,when it is at level of energy minima, the conductivity is zero. Furthermore, in the limit $v_F = 0, \Delta = 0$ Rashba term is disappearing in the action and we have non-relativistic fermions. Then moat absent in the spectrum, we have only outer k_{2F} Fermi momentum and conductivity acquires Drude form

$$\sigma_{xx}(\omega) = \frac{k_{2F}^2}{4\pi m} \frac{i}{\omega + 2i\eta}.$$

We have presented here the calculation of the polarization operator in the fermionic system, which have moat type spectrum. The answer has normal part, leading to Drude conductivity and anomaly part, defined by Rashba SO-term in the Hamiltonian. The result presents correct, expected limits at $v_F = 0$, when Rashba term is zero, at $m = \infty$,when non-relativistic part of the Hamiltonian is zero and we have only Rashba term. At the minimal chemical potential the conductivity became zero.



a

b

Fig.4. (a) Lower branch of spectrum with filled Fermi sea. (b) The Fermi momentum of inner and outer circles

In **third** chapter it is presented the mini-superspace limit of boundary three-point function in Liouville field theory. In this chapter the matrix elements of the boundary Liouville field theory are studied in the mini-superspace limit. In the minisuperspace limit only the zero mode dynamics survives and the theory is reduced to the corresponding quantum mechanical problem.

The mini-superspace limit of the Liouville field theory was considered in some works. In this chapter we study the mini-superspace limit of the boundary three-point function in the BLFT. The conformal invariant action has the form:

$$S=\int_{-\infty}^{\infty}d\tau\int_{0}^{\pi}d\sigma\left(\frac{1}{4\pi}(\partial_{a}\phi)^{2}+\mu e^{2b\phi}\right)+\int_{-\infty}^{\infty}d\tau M_{1}e^{b\phi}\,|_{\sigma=0}+\int_{-\infty}^{\infty}d\tau M_{2}e^{b\phi}\,|_{\sigma=\pi}$$

where M_1 and M_2 are the corresponding boundary cosmological constants.

The boundary three-point function in the BLFT was computed and expressed via double Gamma and double Sine functions. Using known asymptotic properties of the double Gamma and Sine functions, we have shown that in the mini-superspace limit the boundary three-point function can be expressed via the Meijer functions with the unit argument or equivalently via the generalized hypergeometric functions with the unit argument. Also the matrix elements are computed for the Morse potential and have shown that they can be expressed via the generalized hypergeometric functions with the unit argument as well. Using the identities, relating different generalized hypergeometric functions with the unit argument, and matching quantum and classical parameters, we established exact agreement between the mini-superspace limit of the boundary three-point function and the matrix elements for the Morse potential. It is important to note that in the BLFT relation of the boundary cosmological parameter to the corresponding quantum parameter appearing in the boundary one-point function is two-fold due to a sign ambiguity in the choice of the square-root branch. It has been found that to match the mini-superspace limit of the boundary three-point function with the corresponding quantum mechanical matrix element we should use the branch with the negative sign. In the thesis we also show that the passing from one branch to the another one brings to additional factor in the normalization of the wave functions corresponding to the boundary condition changing operators. We would like also to mention that various consequences of the branching of the BLFT parameters earlier were considered in some publications.

In the mini-superspace limit the boundary Liouville field theory is described by the Hamiltonian with the Morse potential:

$$-\frac{\partial^2\psi}{\partial\phi_0^2}+\pi\mu e^{2b\phi_0}\psi+(M_1+M_2)e^{b\phi_0}\psi=k^2b^2\psi$$

The corresponding eigenfuntions satisfy the Schrodinger equation. It has been discussed in this chapter quasi-classical properties of the boundary three-point functions. We found perfect agreement with the corresponding quantum mechanical calculations. The matching of the calculations required to consider the negative branch in the branched correspondence of the classical and quantum parameters. It has been shown that passing from one branch to another leads to the change in the normalization of the wave functions. It has been found the flip of the boundary conditions induced by the exponential operators in the minisuperspace limit.

In this part we also constructed the topological defect in the Liouville field theory producing jump in the value of cosmological constant. The action of the Liouville theory is:

$$S=\frac{1}{2\pi i}\int\left(\partial\phi\bar{\partial}\phi+\mu\pi e^{2b\phi}\right)d^2z.$$

Here we use a complex coordinate $z=\tau+i\sigma$, and $d^2z\equiv dz\wedge d\bar{z}$ is the volume form. The field $\phi(z,\bar{z})$ satisfies the Liouville equation:

$$\partial\bar{\partial}\phi=\pi\mu be^{2b\phi}.$$

The general solution to of last equation can be written in terms of two arbitrary functions $A(z)$ and $B(\bar{z})$:

$$\phi=\frac{1}{2b}\log\left(\frac{1}{\pi\mu b^2}\frac{\partial A(z)\bar{\partial}B(\bar{z})}{(A(z)+B(\bar{z}))^2}\right).$$

Classical expressions for the holomorphic and anti-holomorphic components of the energy-momentum tensor are

$$T=-(\partial\phi)^2+b^{-1}\partial^2\phi,$$
$$\bar{T}=-(\bar{\partial}\phi)^2+b^{-1}\bar{\partial}^2\phi.$$

We construct it using the Cardy-Lewellen equation for the two-point function with defect. It has been shown that there are continuous and discrete families of such kind of defects. For the continuous family of defects we found the Lagrangian description and check its agreement with the solution of the Cardy-Lewellen equation using the heavy asymptotic semiclasscial limit. The connection between the fractional Hall effect and two-dimensional conformal field theories is well known and has many different aspects.Much effort was put into establishing connections between various conformal blocks and Laughlin's wave functions.

Cappelli with collaborators studied the Hall effect using incompressible fluid approach with symmetry. In were considered conformal blocks and their relation to Hall effect on arbitrary Riemann surface. Another very interesting aspect is the relation between edge physics of Hall effect and 2D CFT with boundaries and topological defects. In the recent works emerged important role of 2D Liouville CFT in study of the fractional Hall effect. In these works the Liouville theory cosmological constant plays the role of the chemical potential.

Therefore one can hope that boundary Liouville field theory and Liouville theory with defects can have applications to the edge physics of Hall effect. In the fourth chapter it is studied three-point boundary correlation function in the Liouville field theory. The motivation for the second work was the fact that in the FQHE one has jump of the chemical potential .In the fifth chapter it is constructed topological defect gluing Liouville field theories with different cosmological constant. We have an impression that our construction can be useful to understand the physics of the mentioned jump. It has been checked that the system of the defect equations of motion guarantees that both components of the energy-momentum tensor are continuous across the defects and therefore describes topological defects.

In **conclusion**, the main results of the dissertation are listed:

1. It has been obtained current-current correlation function in 3D massive Dirac theory with chemical potential.
2. It has been shown the behaviour of current-current correlation function in third order of Feynman diagrams in the presence of chemical potential and magnetic field.
3. It has been studied the moat spectra of topological insulator regarding cold atoms with spin-orbit interacting.
4. It has been obtained boundary three point function on mini-superspace in Liouville field theory and also has been computed matrix elements for the Morse potential quantum mechanics. An exact agreement between the former and latter has been

found. We show that both of them are given by the generalized hypergeometric functions.
5. We construct topological defects in the Liouville filed theory producing jump in the value of cosmological constant. We construct them using the Cardy-Lewellen equation for the two-point function with defect.
6. We show that there are continuous and discrete families of such kind of defects.
For the continuous family of defects we also find the Lagrangian description and check its agreement with the solution of the Cardy-Lewellen equation using the heavy asymptotic quasi-classical limit.

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Հոլի քվանտային երևույթի որոշ ասպեկտները
Ամփոփում

Դիսերտացիայի մեջ ուսումնասիրվում են Հոլլի քվանտային երևույթի որոշ ասպեկտները: Դիտարկվել է 2+1 չափանի Դիրակի անզանգված ֆերմիոններից կազմված համակարգ, որի օրինակ է գրաֆենը:Հաշվվել է հաղորդականությունը գրաֆենում կամայական քիմիական պոտենցիալի դեպքում Ֆեյնմանի դիագրամի մեկ օղակային մոտավորությամբ: Նման համակարգերը հետաքրքիր են նրանով, որ այդ մոդելներում հնարավոր է դիտել Հոլլի քվանտային երևույթ մագնիսական դաշտի բացակայության դեպքում: Աշխատանքի մեջ ստացվել է արտահայտություն բևեռացման օպերատորի համար, որի հաշվարկը թույլ է տալիս որոշել նույն հաղորդականությունը:

Դիսերտացիայի մեջ հաշվվել է նաև 2+1 չափանի համակարգի համար հոսանք-հոսանք կորելիացիոն ֆունկցիան կամայական քիմիական պոտենցիալի և դասական արտաքին մագնիսական դաշտի առկայությամբ: Հաշվարկներն իրականացվել են Ֆեյնմանի դիագրամների երրորդ կարգի մոտավորությամբ (3+1 չափանի համակարգերում նման դիագրամներն անհիհլացվում են Ֆարրիի թեորեմի համաձայն):

Ժամանակակից ֆիզիկայում կան որոշ նյութեր, ինչպիսիք են օրինակ տոպոլոգիական մեկուսիչներն եզրային վիճակներով, որոնք մեծ հետաքրքրություն են ներկայացնում: Տոպոլոգիական մեկուսիչների համար հաշվվել է բևեռացման օպերատորը ոչ-ռելյատիվիստիկ ֆերմիոնների Ռաշբա սպին-ուղեծրային փոխազդեցության առկայության դեպքում:Այդպիսի ֆերմիոնների էներգիական սպետրը փոսաձև է և ունի մինիմում շրջանի վրա, ի հակադրություն Դիրակի կամ ոչ-ռելյատիվիստիկ ֆերմիոնների այստեղ Ֆերմի «ծովն» ունի Կորբինոյի սկավառակի երկրաչափություն, որն արտահայտում է խոտուրումների տեղափոխական հատկությունները:

Դիտարկվել են տոպոլոգիական դեֆեկտներ Լիուվիլլի դաշտի տեսության շրջանակներում կոսմոլոգիական հաստատունի թռիչքով: Նման մոդել կառուցվել է օգտագործելով Կարդի-Լևիլենի հավասարումը դեֆեկտով երկու կետանի ֆունկցիայի համար:Ցույց է տրվել,որ կան նման դեֆեկտների դիսկրետ և անընդհատ ընտանիքներ:Դեֆեկտների անընդհատ ընտանիքի համար գտնվել է Լագրանժիան նկարագրությունը և ստուգվել է համաձայնությունը Կարդի-Լևիլենի հավասարման հետ՝ ծանր ասիմպտոտիկ կիսադասական սահմանում:Կապը Հոլլի քվանտային երևույթի և երկու չափանի կոնֆորմ դաշտի տեսության հետ հայտնի և ունի տարբեր ասպեկտներ:

ՈՒսումնասիրվել են եզրային Լիուվիլի տեսության մատրիցական էլեմենտները

մինի-գերտարածական սահմանում, այդ սահմանում մնում են միայն զրոյական մոդերը, որի արդյունքում գալիս ենք քվանտամեխանիկական խնդրի: Մինի-գերտարածական սահմանում հաշվվել է եզրային Լիուվիլլի դաշտի տեսության երեք կետանի ֆունկցիան, որն արտահայտվել է զույգ Գամմա և սինուսոիդային ֆունկցիաների միջոցով: Օգտագործելով ասիմպտոտիկ հատկությունները նշված ֆունկցիաների՝ ցույց է տրվել, որ մինի-գերտարածական սահմանում եզրային երեք կետանի ֆունկցիան կարող է արտահայտվել Մեյերի միավոր արգումենտով կամ էկվիվալենտ ընդհանրացված միավոր արգումենտով հիպերերկրաչափական ֆունկցիաների միջոցով: Հաշվվել են նաև Մորզե պոտենցիալի մատրիցական էլեմենտները և ցույց է տրվել,որ նրանք կարող են արտահայտվել համապատասխան հիպերերկրաչափական ֆունկցիաներով:

Некоторые аспекты явления квантового Холла
Резюме

В диссертации изучаются некоторые аспекты квантового эффекта Холла.Была рассмотрена (2+1)-мерная система состоящая из дираковских безмассовых фермионов,примером которого является графен. Проводимость была рассчитана для графена при произвольном химическом потенциале в приближении однопетлевой диаграммы Фейнмана. Такие системы интересны тем, что в этих моделях можно наблюдать явление квантового эффекта Холла при отсутствии нулевого магнитного поля. В работе была получена формула для поляризационного оператора, расчет которой позволяет определить проводимость. В диссертации рассчитана также ток-ток корреляционная функция для 2+1-мерной системы в случае произвольного химического потенциала и неквантованного внешнего магнитного поля. Расчеты проводились в приближении третьего порядка диаграмм Фейнмана (в 3 + 1 мерных системах такие диаграммы не учитываются согласно теореме Фарри). В современной физике существуют некоторые вещества, например топологические изоляторы с конечными состояниями, которые представляют большой интерес. Для топологических изоляторов рассчитан оператор поляризации для спин-орбитального взаимодействия Рашбы с нерелятивистскими фермионами. Энергетический спектр таких фермионов представляет собой яму и имеет минимальную окружность. В отличие от дираковских или нерелятивистских фермионов, море Ферми имеет геометрию диска Корбино, которая отражает транспортные свойства возбуждений. Рассмотрены топологические дефекты в теории Лиувилля с различными космологическими константами. Аналогичная модель была построена с использованием уравнения Карди-Левилена с дефектом для двухточечной функции. Показано, что существуют дискретные и постоянные семейства с такими дефектами. Для непрерывного семейства дефектов было найдено описание лагранжиана и показано согласие с уравнением Карди-Левилена в тяжелом квазиклассическом пределе. Взаимосвязь между квантовым эффектом Холла и двумерной комформной теорией поля имеет различные аспекты. Математические элементы граничной теории Лиувилля изучались в рамках мини-суперпространстве. В этом пределе остаются только нулевые моды, и в результате мы приходим к квантово-механической задаче. Матрич-ные элементы граничной теории Лиувилля изучались в мини-суперпространств, где остаются только нулевые моды, и в результате мы приходим к квантово-механической задаче. В мини-суперпространстве была рассчитана предельная

трехточечная функция теории поля Лиувилля, которая была выражена через четные Гамма-функции и синусоидальные функции. Используя асимптотические свойства этих функций, было показано, что конечная трехточечная функция в пределе мини-суперпространства может быть выражена через функцию Мейера единичным аргументом или через эквивалентную обобщенную гипергеометрическую функцию единичным аргументом. Матричные элементы потенциала Морзе также были вычислены,показано,что они могут быть выражены в соответствующих гипергеометрических функциях.